PLAIN TRIGONOMETRY

Rendered *Easy* and *Familiar*,

By CALCULATIONS in ARITHMETICK only:

WITH ITS

APPLICATION and USE

In ascertaining all Kinds of HEIGHTS, DEPTHS, and DISTANCES,

IN THE

HEAVENS, as well as on the EARTH and SEAS;

WHETHER OF

Towers, Forts, Trees, Pyramids, Columns, Wells, Ships, Hills, Clouds, Thunder and *Lightning, Atmosphere, Sun, Moon, Mountains* in the *Moon, Shadows* of *Earth* and *Moon, Beginning* and *End* of *Eclipses*, &c.

In which is also shewn,

A Curious *Trigonometrical Method* of discovering the Places where BEES hive in large WOODS, in order to obtain, more readily, the *salutary Produce* of those *little Insects*.

By the Rev. Mr. TURNER, late of *Magdalen-Hall, Oxford*,
Author of The *View* of the *Earth*;—*View* of the *Heavens*;—*System* of *Gauging*;—and *Chronologer Perpetual*.

Cuncta Trigonus *habet, satagitque docta* Mathesis,
Ille *aperit clausum, quicquid* Olympus *habet*.

Within the grand Triangle *lies unveil'd*,
What Sages *sought for, and what* Heaven *conceal'd*.

LONDON.
Printed for S. CROWDER, in Pater-noster-Row; and S. GAMIDGE, Bookseller, in Worcester. MDCCLXV.

TO THOSE

GENTLEMEN,

Whoſe *Genius* may incline, or *Employment* lead them to the Study of the

MATHEMATICKS.

GENTLEMEN,

TRIGONOMETRY has always been look'd on as one of the moſt uſeful Branches of *Mathematical* Learning. *Navigation*, *Surveying*, *Aſtronomy*, &c. ſtand wholly upon this *Baſis*. But the common Method of anſwering theſe Problems being by large Tables of *Sines*, *Tangents*, and *Secants*, renders it not only expenſive by the purchaſe of them; but often precarious in the Solution, by the Miſtakes of the Preſs. I have therefore, for the Uſe of the *Young Mathematician*, (from a Conſideration of what has been publiſhed on this curious Subject) compoſed the preſent *Syſtem*, by which any of the *Caſes* in *Right* or *Oblique* Plain Triangles may be anſwered on the *Spot*, by an eaſy Calculation in *Arithmetick* only. The great Advantages reſulting from this Method to *Gentlemen* in the *Army* or *Navy*, as well as to thoſe in their *private Studies* at *Home*, muſt immediately appear; as it will be found to anſwer the moſt neceſſary Problems as *expeditiouſly* as *Logarithms*; and at the ſame Time wholly deliver you from thoſe *voluminous Tables* and the *inartificial Fatigues* of carrying them always with you.--- Should this little Treatiſe be ſo happy as to meet your Approbation, it will give a particular Pleaſure to,

Your moſt humble Servant,

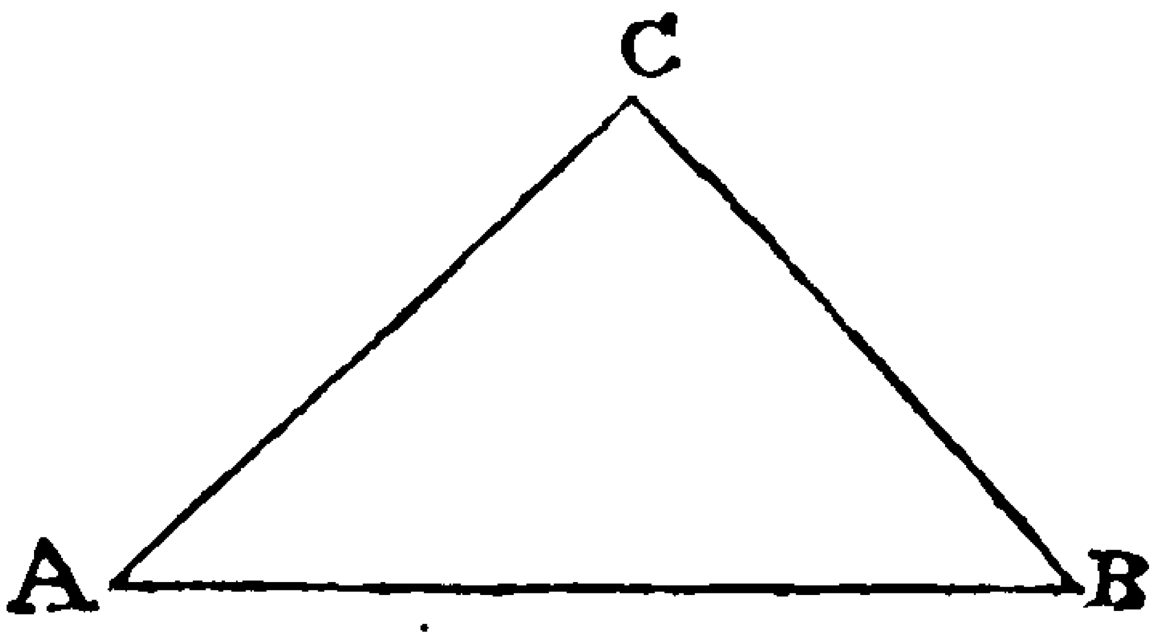
The AUTHOR.

PLAIN

TRIGONOMETRY.

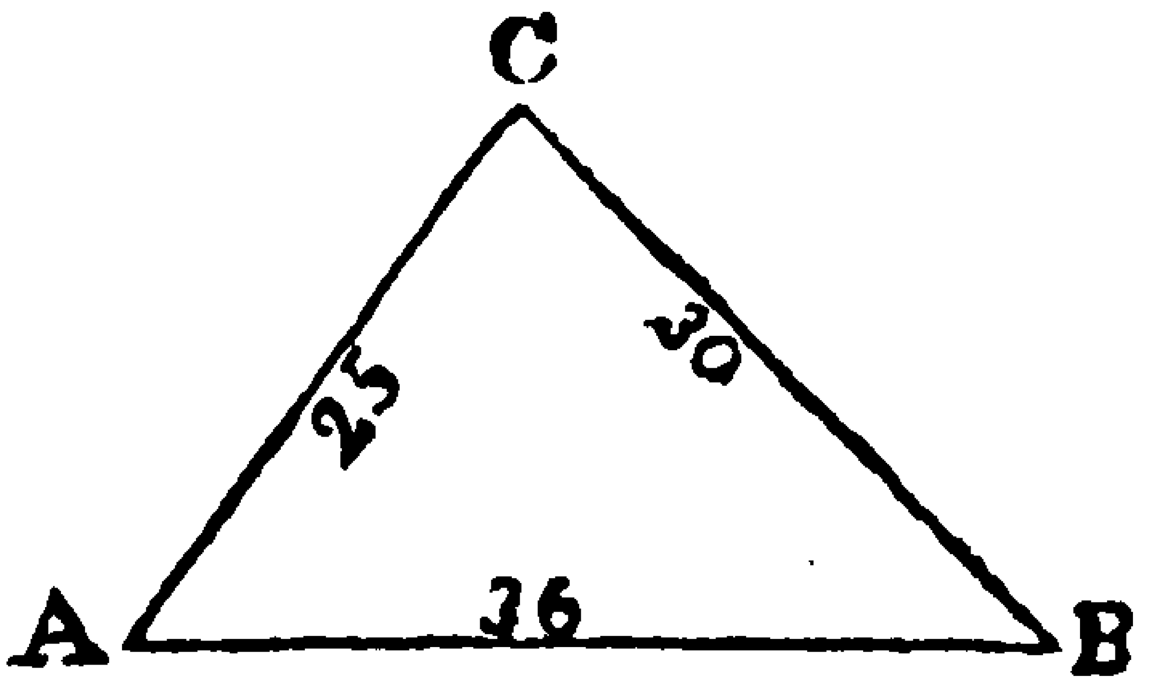
TRIGONOMETRY is that Part of *Mathematicks*, which is employed in calculating the *Sides* and finding the *Angles* of any *Triangle* required; it is of the greateſt *Uſe* in Life, as nothing in *Navigation*, *Aſtronomy*, &c. can be done without it; and depends on the Knowledge of the following Obſervations.

(1ſt.) Every Triangle conſiſts of Six Parts; that is, of *Three Sides* and *Three Angles*, as in the Figure ABC; the Three Sides are, AB, AC, CB, and the Three Angles, A, B, C.



NOTE. Sometimes an *Angle* is expreſſed by *Three* Letters; in that Caſe, the *Middle* Letter denotes the *Angular Point*. Thus, ABC expreſſes the Angle B; BAC the Angle A; and ACB the Angle C.

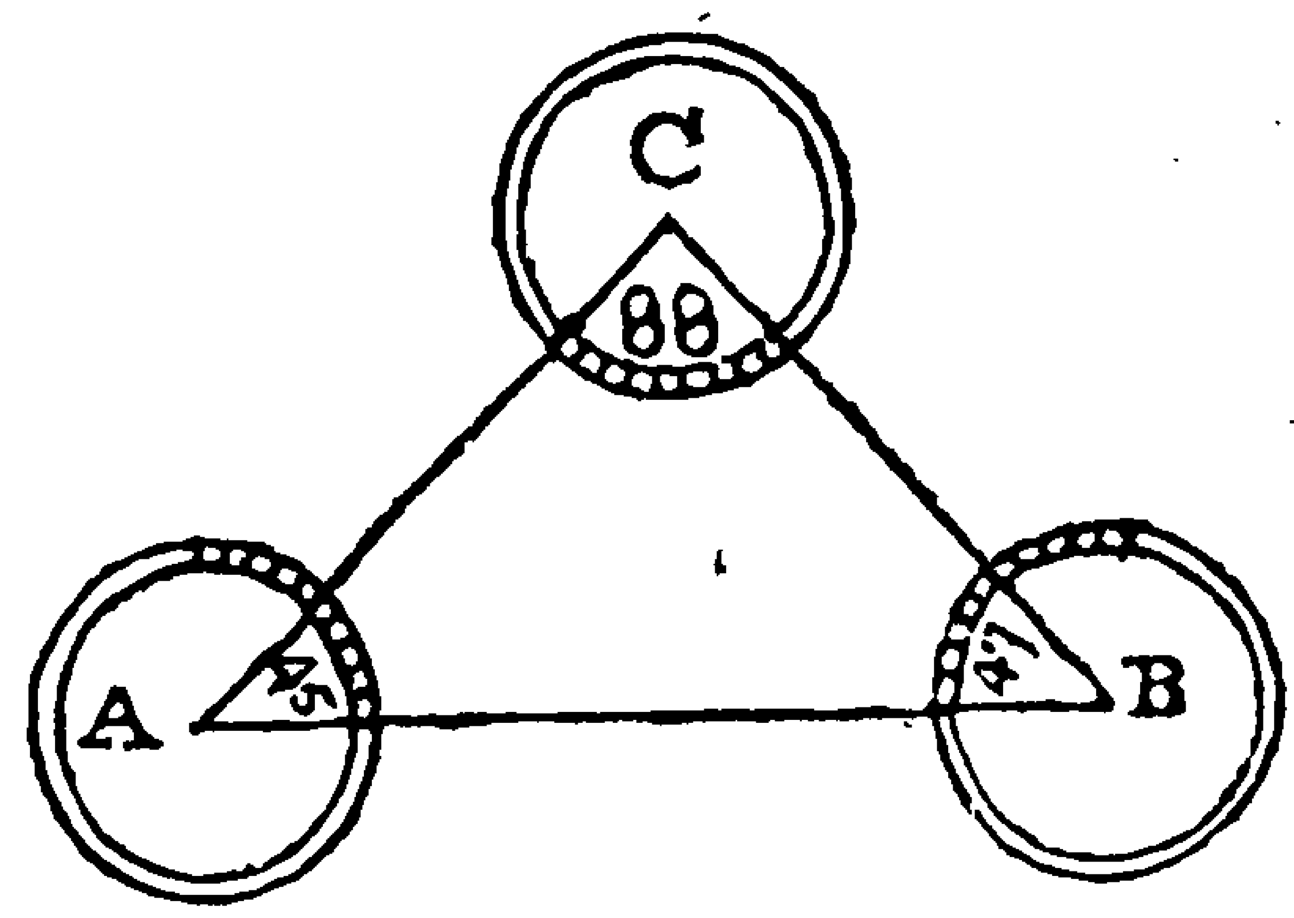
(2d.) The *Sides* of all plain Triangles are meaſured by a Line of equal *Parts*, as of *Inches*,—*Feet*,—*Yards*,—or *Leagues*.



Thus, The Side AB is 36 Leagues.—The Side AC 25 Leagues.—And the Side BC is 30 Leagues.

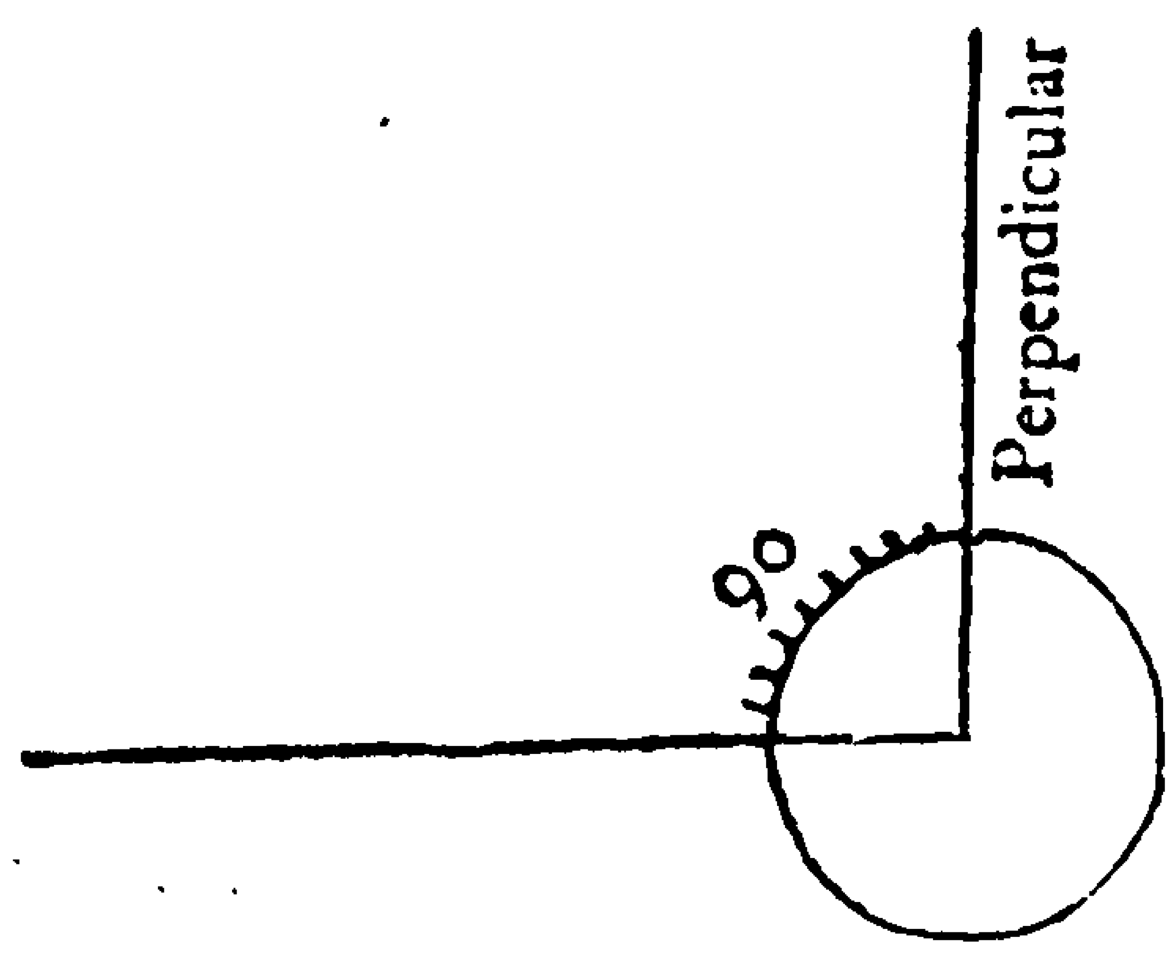
(3d.) The

(3d.) The *Angles* are meaſured by the *Arch* of a *Circle* deſcribed upon the Angular Point, and contained between the Two Legs that form the Angle.

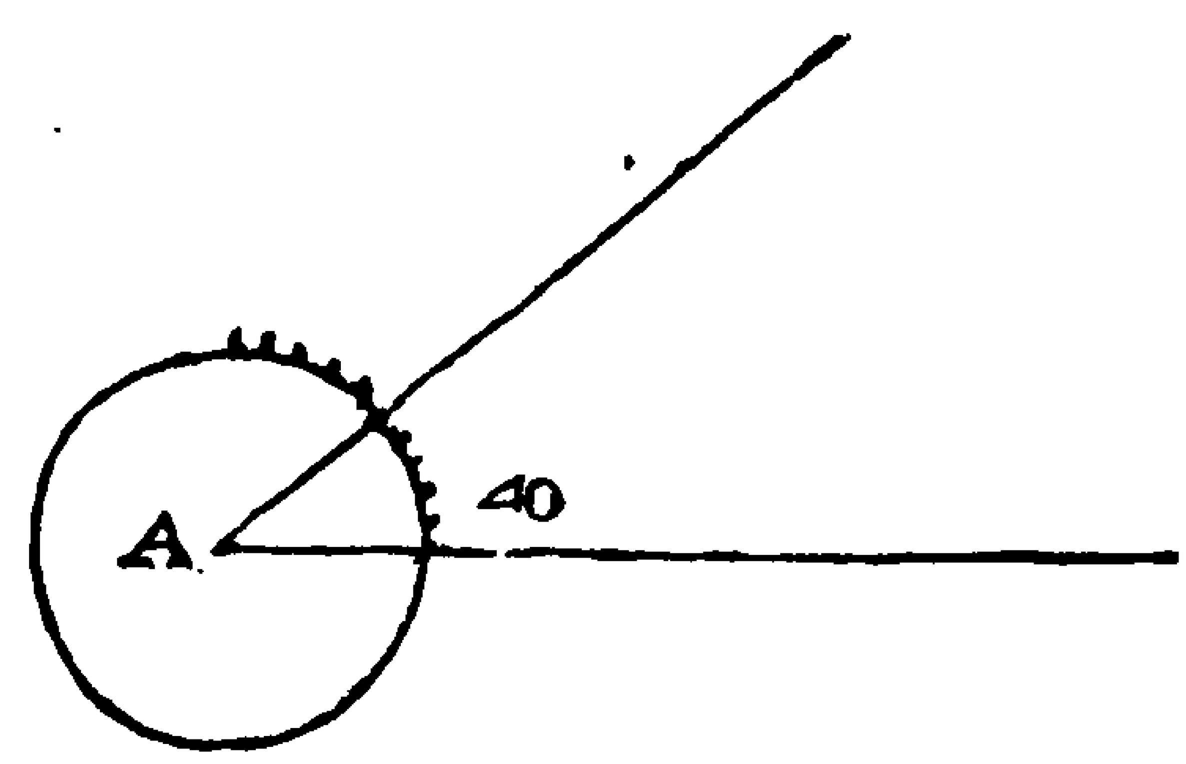


Note. Every Circle is divided into 360 equal Parts, called *Degrees*; each of which are divided into 60 more, called *Minutes*: And the Number of Degrees contained between the Two Legs, that conſtitute the Angle, is the *Meaſure* of that Angle. Thus, The Angle A is 45 *Degrees.*—The Angle B, 47.—The Angle C, 88.

(4th.) If the Arch of a Circle intercepted between the Two Legs be exactly 90 Degrees, the Angle is called a *Right Angle*, and the Legs are *perpendicular* to one another.

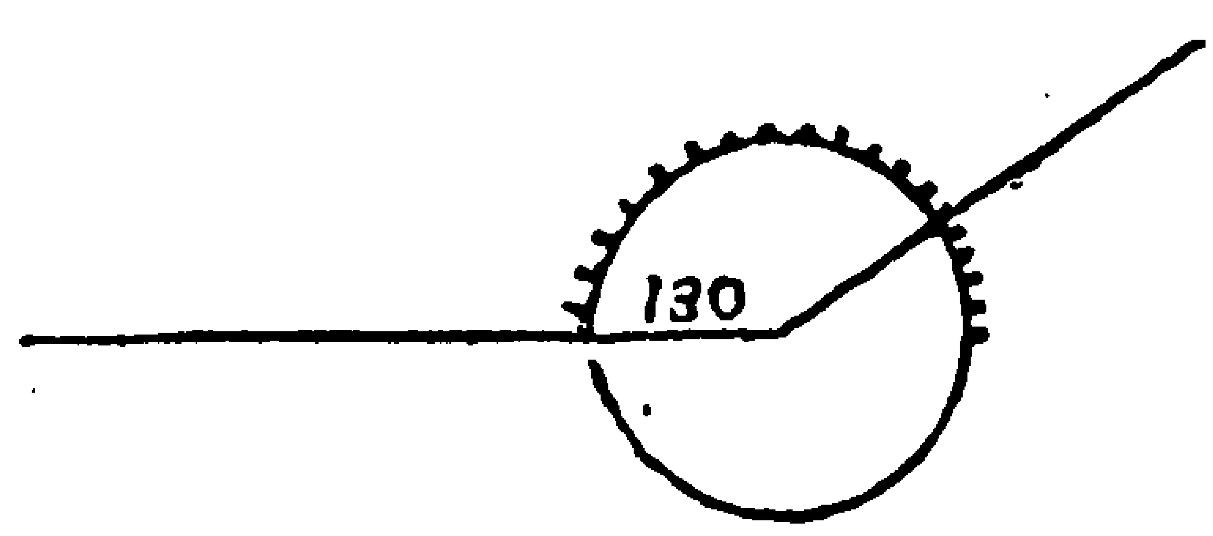


(5th.) If the Arch of the Circle be *leſs* than 90 Degrees, the Angle is ſaid to be *Acute*.



Note. What an *Acute* Angle wants of 90 Degrees, is called the *Complement* of that Angle. Thus, Suppoſe the Angle A was 40 Degrees; then its Complement is 50 Degrees; for 40 added to 50 make 90, as obſerved before.

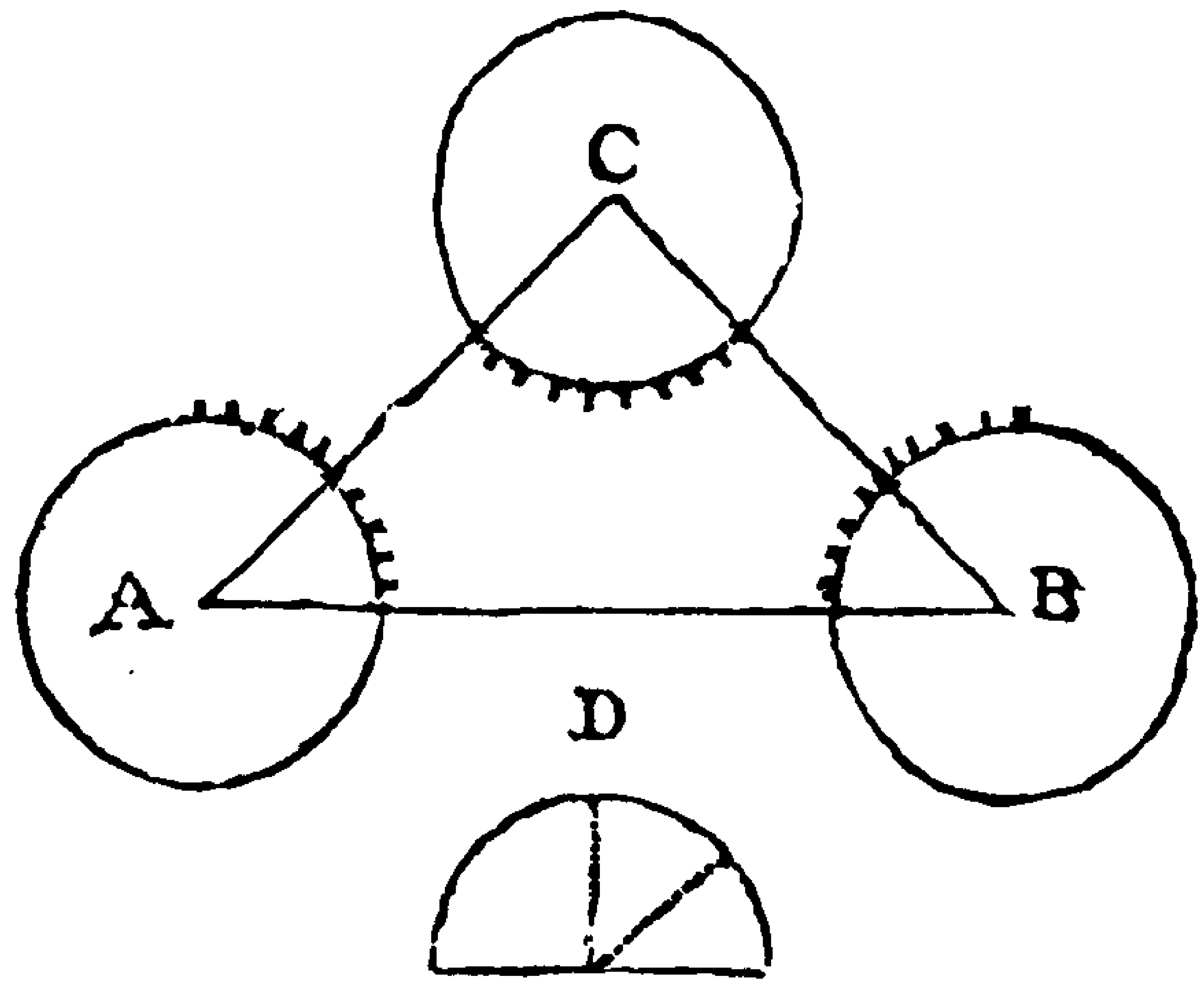
(6th.) If the Arch of a circle be *more* than 90 Degrees, the Angle is ſaid to be *Obtuſe*; and ſo continues to 180 Degrees, where the Angle vaniſhes, the Lines becoming *Strait*.



Note. What an *Obtuſe* Angle wants of 180 Degrees, is alſo called the *Complement of that Angle to a Semicircle*.

(7th.) The

(7th.) The Three Angles of every plain Triangle, being taken together, make 180 Degrees (equal to a Semicircle), and this they always do, let the Triangle be drawn however you please.

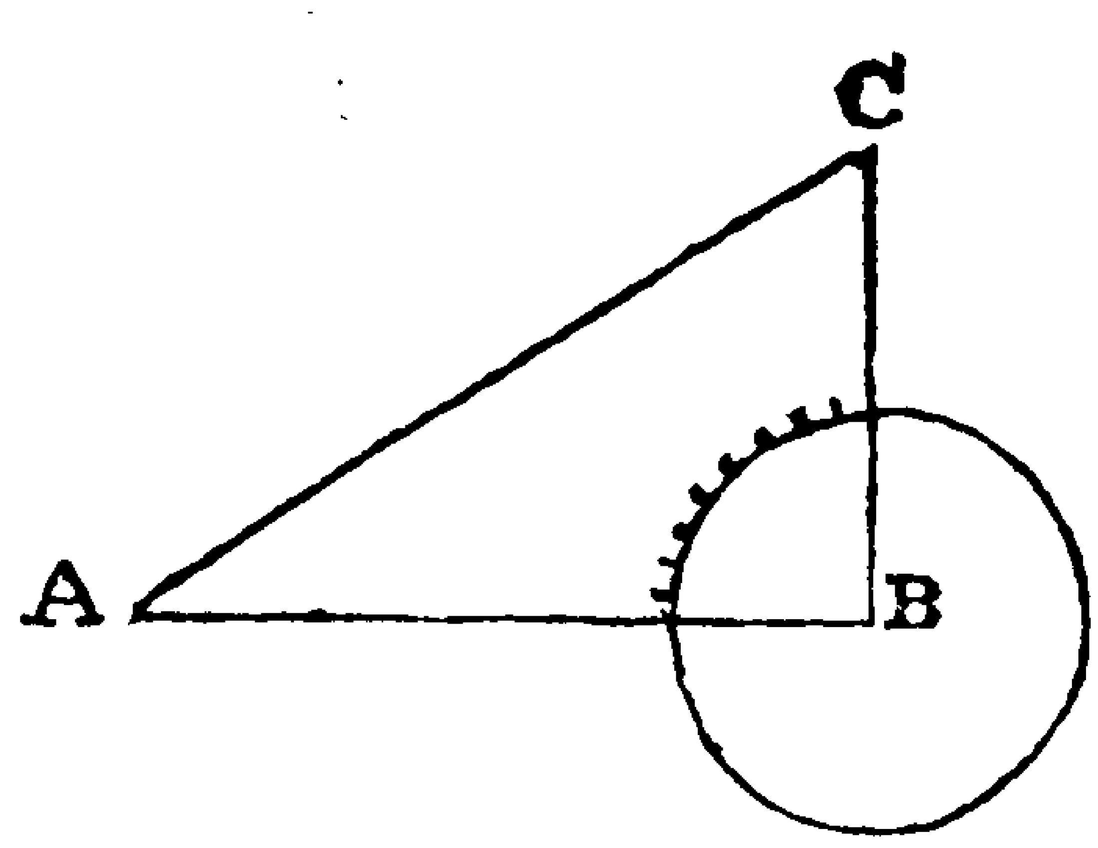


Thus, If the *Semicircle* D be drawn with the same *Radius*, or opening of the Dividers, as the little Circles on the Angles A, B, C, are; you will find, by taking off the several Arches, and applying them to the Semicircle, that they will just fill it up, and thereby make 180 Degrees; because every Semicircle contains that Number of Degrees.

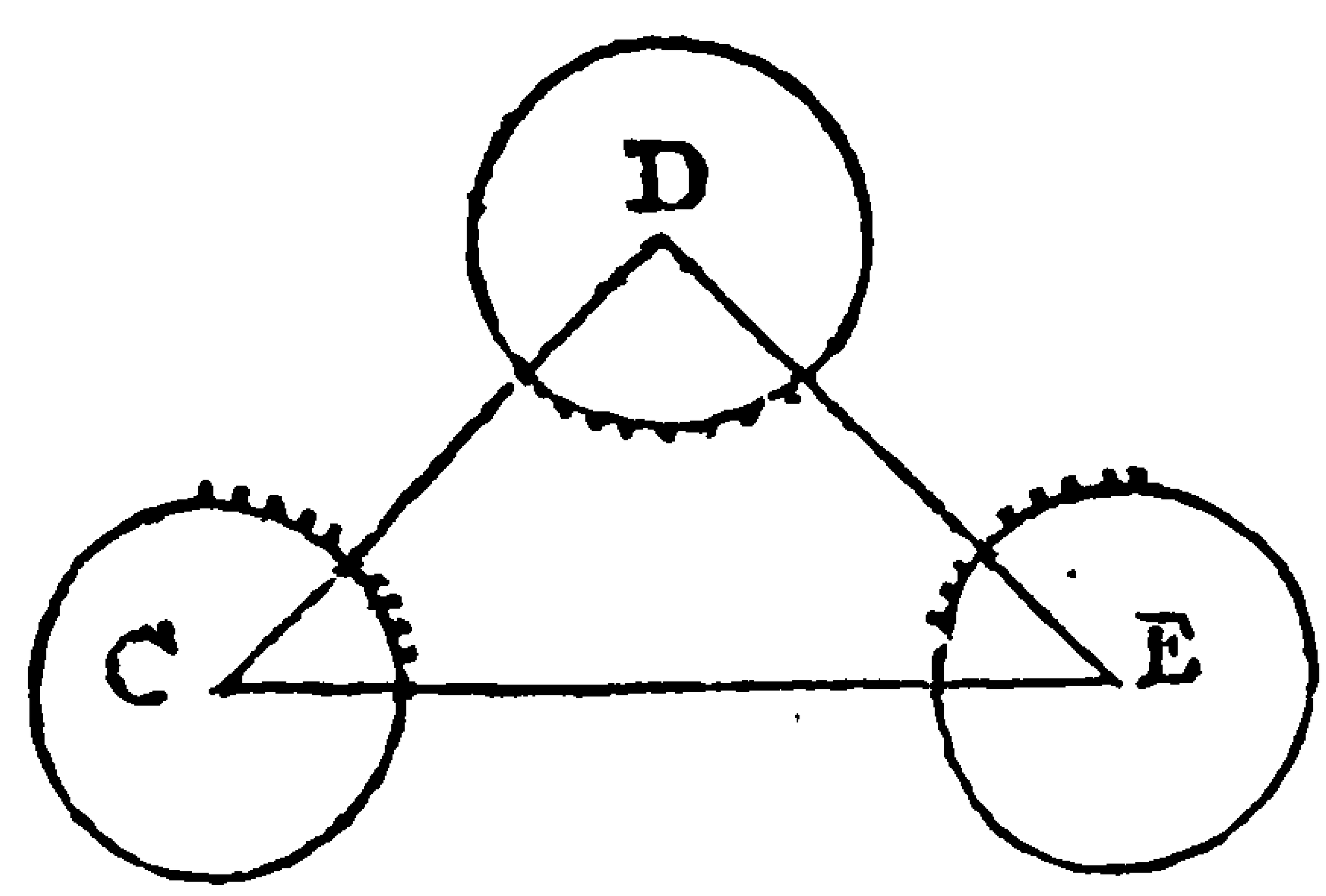
Hence it is evident, that if one Angle be a *Right One*, the other Two will be *Acute*; and taken together, be equal to one *Right Angle*, or just 90 Degrees.

Hence also, if *Two* Angles of any Triangle are known, the *Third is* easily found, being only the *Degrees* the other Two Angles want of 180.

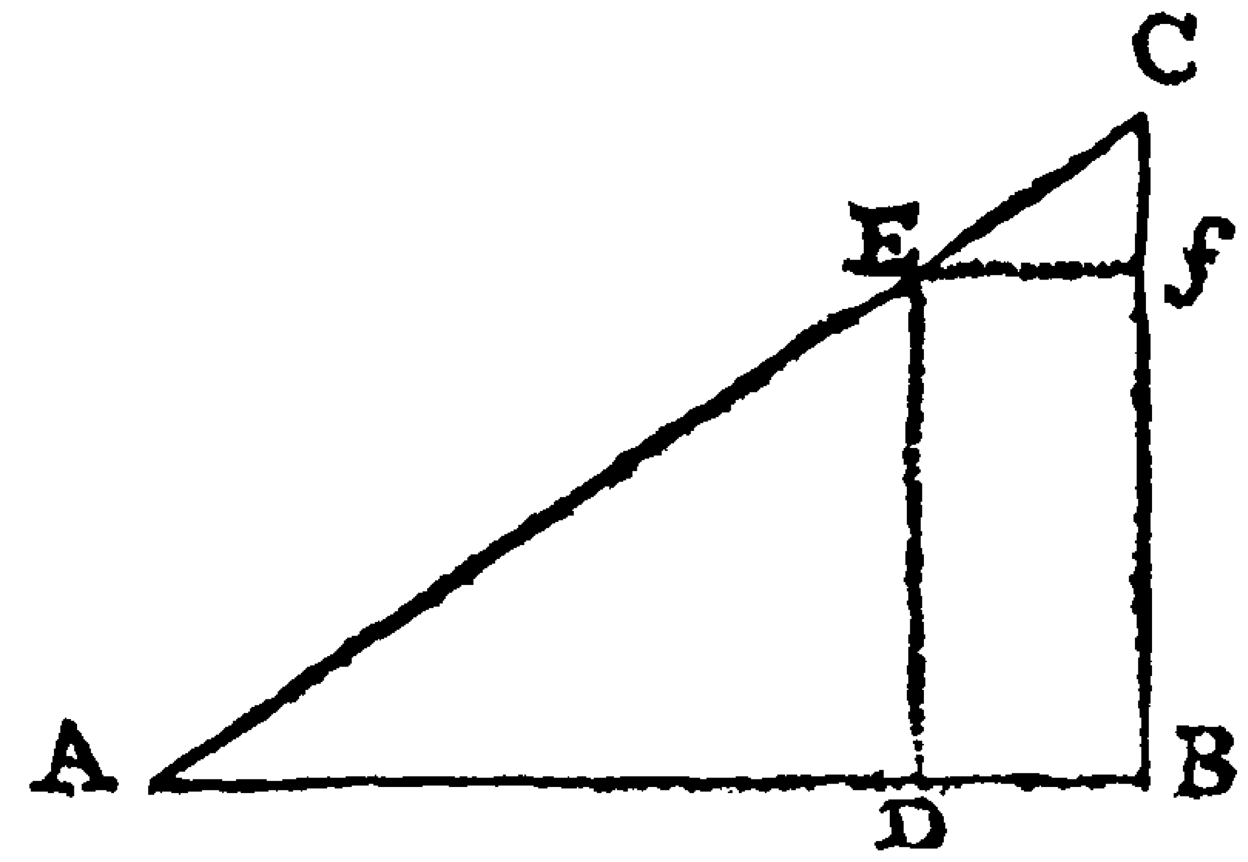
(8th.) If a *Triangle* has one *Right Angle*, it is called a *Right Angled Triangle*: Thus ABC is a Right Angle Triangle, Right-angled at B.—In all Right Angle Triangles, the longest Leg is called the *Hypothenuse*;—the Leg on which it stands, the *Base*;—and the other Leg, the *Perpendicular*.



(9th.) If neither of the Angles is a Right One, then it is called an *Oblique Angled Triangle*; as the Triangle CDE is an *Oblique Triangle*.



(10th.) If



(10th.) If the Angles of one Triangle are equal to the Angles of another Triangle, the Sides of the former are proportional to the Sides of the latter.

Thus, If in the Triangle ABC, you draw the Line ED parallel to CB, the ſmaller Triangle ADE will be *ſimilar to*, i. e. will have the ſame Angles with the larger Triangle ABC.—It will therefore always hold, as

AB : AD : : BC : DE.
Or, as AB : BC : : AD : DE.
Or, as AD : DE : : AB : AC.
Or, as AD : DE : : E*f* : *f*C, *&c.*

(11th.) In all Triangles the *greateſt Side* is oppoſite to the *greateſt Angle*; and on the contrary, the *greateſt Angle* is oppoſite the *greateſt Side.*—If Two *Sides* are *equal*, the oppoſite *Angles* are *equal.*—If all the Sides are equal, then all the Angles are equal to each other.

(12th.) In all Triangles, every Side is in proportion to its oppoſite Angle, and every Angle to its oppoſite Side: And further, as the Angle oppoſite to one Side, is to the Angle oppoſite the other Side, ſo are the Sides themſelves to one another; and the contrary, the Sides to the Angles.

Every *Triangle*, as I obſerved before, conſiſts of Six Parts—*Three Sides* and *Three Angles*. If any Three of the Six Parts (excepting the Three Angles) are given, any one, or every one of the reſt may be found, without the painful Deductions and voluminous Tables of *Logarithms*, *Sines*, *Tangents*, and *Secants*, by the following *Rules* and *Axioms* *.

* This *Method* will be found as exact, as that by the *Logarithms*, if you carry on the *Operation* to *Three* or *Four Decimal* Places; but for common Purpoſes, *One* or *Two Decimal* Places will be near enough. You muſt alſo remember to reduce the *Minutes* of the Angles to *Decimals* of a Degree, which is eaſily done, by allowing One tenth for every Six Minutes.——Or you may turn the Minutes into Decimals thus: As 60, the Minutes in one Degree, : are to the Minutes given, : : ſo are 10, 100, 1000, *&c.* to the *Decimal* required.

Of

Of Right Angled TRIANGLES.

THERE are generally reckoned by Writers on this Subject Seven Cases; but by this Method they are all reduced to Four; the Solutions of which depend on the following *Axioms*.

AXIOM I. Divide 4 Times the Square of the Complement of the Angle, whose opposite Side is either given or sought, by 300 added to 3 Times the said Complement; this Quotient added to the said Angle, will give you an *Artificial Number*, called sometimes the *Natural Radius* *, which will ever bear the same Proportion to the Hypothenuse, as that Angle bears to its opposite Side.——In Angles under 45 Degrees, the *Artificial Number* may be found easier thus: Divide 3 Times the Square of the Angle itself, whose opposite side is given or sought, by 1000; the Quotient added to 57.3 †, a fixed Number, that Sum will be the *Artificial Number* required.—*This is to be used, when the* Angles *and a* Side *are given, to find* another Side.

AXIOM II. The Square of both the Legs, *i. e.* the Square of the *Base* and *Perpendicular* added together, is equal to the Square of the *Hypothenuse*; whose Root is the *Hypothenuse* itself.—*This is made use of, when the* Base *and* Perpendicular *are given, to find the* Hypothenuse.

AXIOM III. The *Sum* of the *Hypothenuse* and *One* of the *Legs* multiplied by their *Difference*, the *Square Root* of that Product will be the *other Leg* required.—*This comes into use, when the* Hypothenuse *and* One Leg *is given, to find the* other Leg.

AXIOM IV. *Half* the *Longer* of the *Two Legs*, added to the *Hypothenuse*, is always in Proportion to 86 ‡, as the *Shorter Leg* is to its *opposite Angle*. —*This is useful, when the* Sides *are given, to find the* Angles.

NOTE. These 4 *Axioms* will answer all the Cases of *Right* and *Oblique Angled Triangles*, except the last Case in Obliques, which will require some further Assistance, and will be shown when we come to treat of that Case.

* The *Natural Radius* is only turning the *Right Angle*, = 90 Degrees, into an *artificial Number*, which shall always bear the same Proportion to the *Hypothenuse*, as the given Angle does to its opposite Leg.

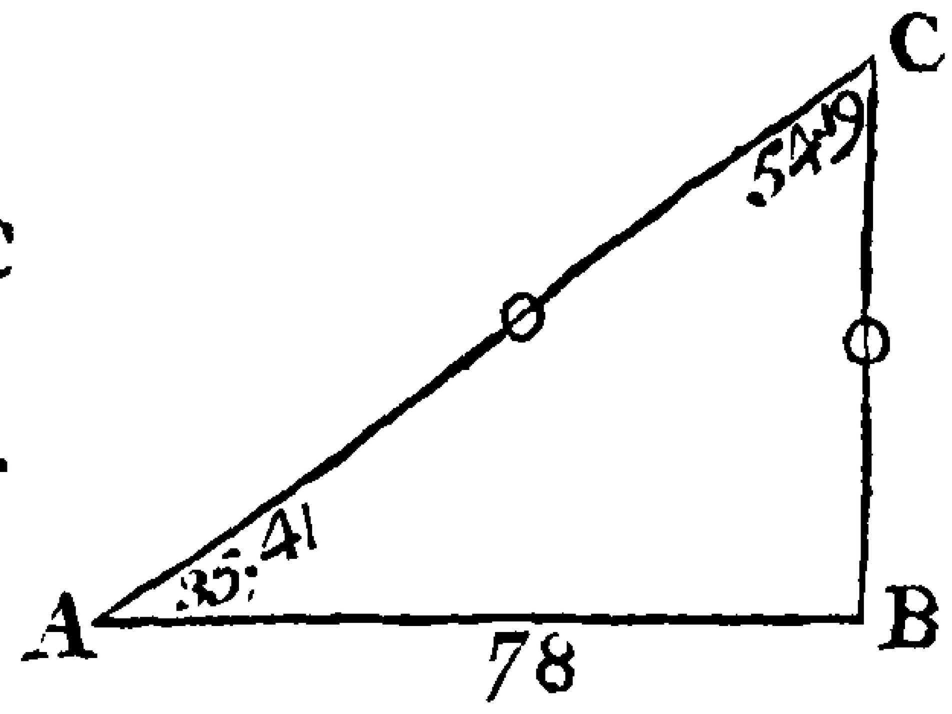
† 57.3 is the *Radius* of a Circle whose Circumference is 360.

‡ 86 = *Radius and Half* of a Circle whose Circumference is 360.

CASE I.

The Acute Angles, and one Leg given; to find the Hypothenuſe and the other Leg.

Given		to find	
	Angle A = 35.41 Angle C = 54.19 Baſe AB = 78		Hypothen. AC and Perpendic. BC.



(1ſt.) Find the Natural Radius by Axiom I.

35.7
35.7
——
2499
1785
1071
——
1274.49
4
——
407.1)5097.96(12.5
4071 (54.3
——
10269 66.8 Natural Radius
8142
——
21276
20355
——
921

35.7
3
——
107.1
300.
——
407.1

(2d.) Find the Hypothenuſe by Axiom I.

Angle C : Baſe : : Nat. Rad.
As 54.3 —— 78 —— 66.8
78
——
5344
4676
——
54.3)5210.4(95.9 + Hypothenuſe
4887
——
3234
2715
——
5190
4887
——
303

(3d.) Find the Perpendicular by Axiom III.

96 To Hypothenuſe
78 Add the Baſe
——
174 Sum multiply
18 by Difference
——
1392
174
——
Extract the Root 3132(55.9 + Perpendicular
25
——
105)632
525
——
1109)10700
9981
——
719

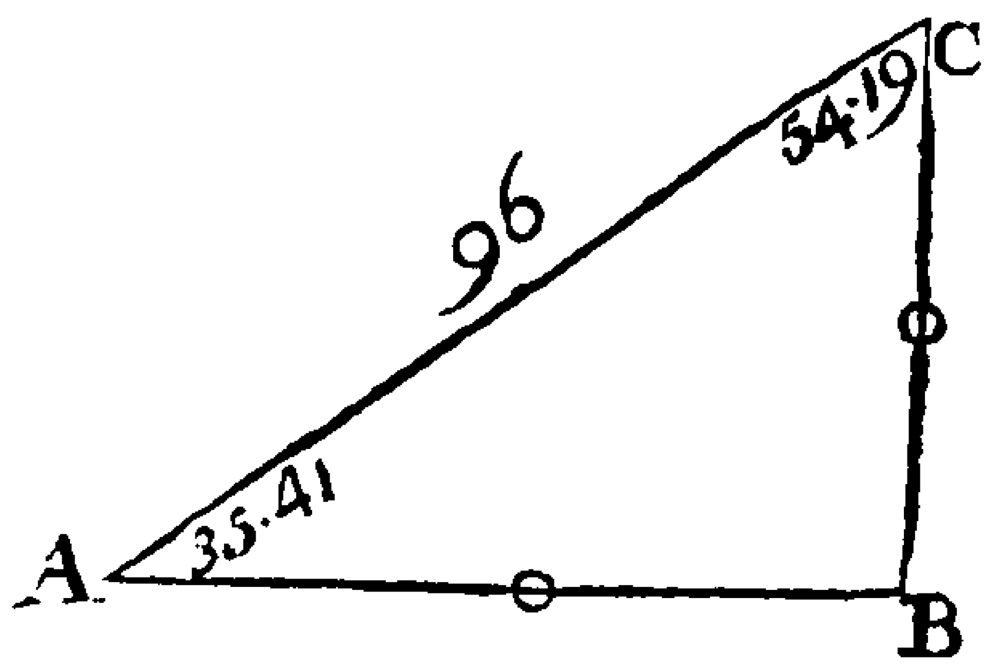
Anſwer, { Hypothenuſe, 95.9 +, or 96.
Perpendicular, 55.9 +, or 56.

CASE

CASE II.

The Hypothenuſe and Angles given, to find the Two Legs.

Given		to find	
	Hypoth. AC = 96		Perpendic. CB
	Angle A = 35.41		and
	Angle C = 54.19		Baſe AB.



(1ſt.) Find the Natural Radius by Axiom I.

35.7
35.7
2499
1785
1071
1274.49
3
1.000)3.823.47
57.3 Add
61.1 Natural Radius

(2d.) Find the Perpendicular by Axiom I.

Nat. Rad. : Hypoth. :: Angle A
As 61.1 —— 96 —— 35.7
96
2142
3213
61.1)3427.2(56 Perpen
3055
3722
3666
56

(3d.) Find the Baſe by Axiom III.

96 To Hypothenuſe
56 Add Perpendicular
152 Sum multiplied by
40 The Difference
Extract the Root 6080(77.9 + Baſe
49
147)1180
1029
1547)15100
13941
1169

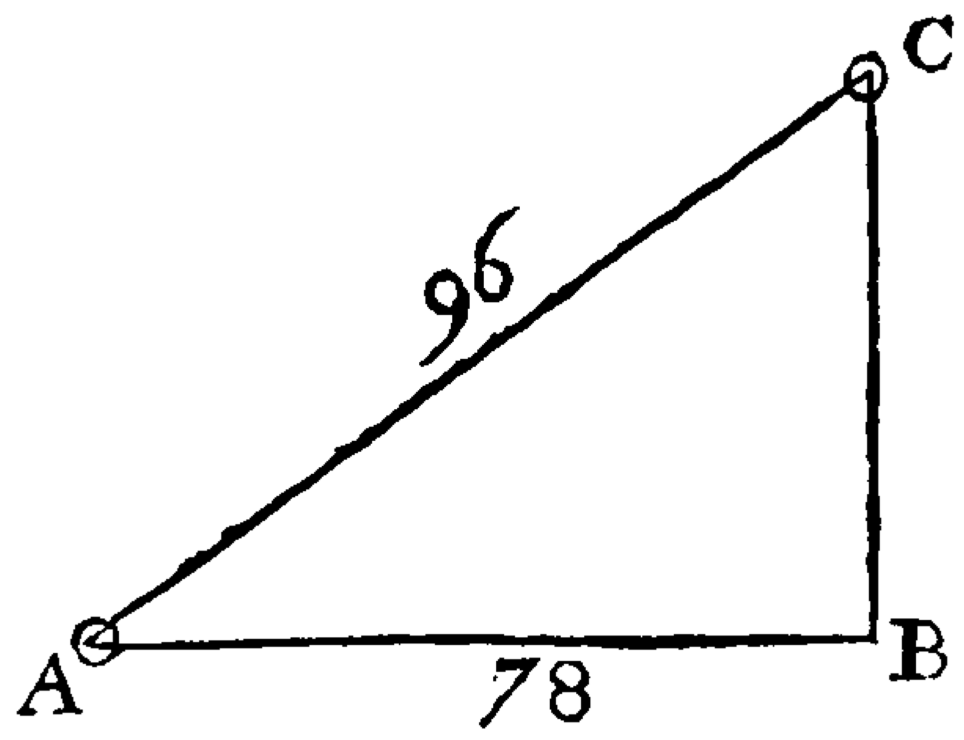
Anſwer, { Perpendicular, 56.
Baſe, 77.9 +, or 78. }

CASE

CASE III.

The Hypothenuſe and One Leg given, to find the Angles and the other Leg.

Given $\left\{ \begin{array}{l} \text{Baſe AB} = 78 \\ \text{Hypoth. AC} = 96 \end{array} \right\}$ to find $\left\{ \begin{array}{l} \text{Perpendic. CB} \\ \text{and} \\ \text{Angles A \& C.} \end{array} \right.$



(1ſt.) Find the Perpendicular by Axiom III.

```
               96  To Hypothenuſe
               78  Add the Baſe
               --
              174  Sum multiply
               18  By Difference
              ---
             1392
             174
             ----
Extract the Root 3132(55,9 + Perpendicular
             25
             --
         105)632
             525
             ---
       1109)10700
             9981
             ----
              719
```

(2d.) Find the Angle by Axiom IV.

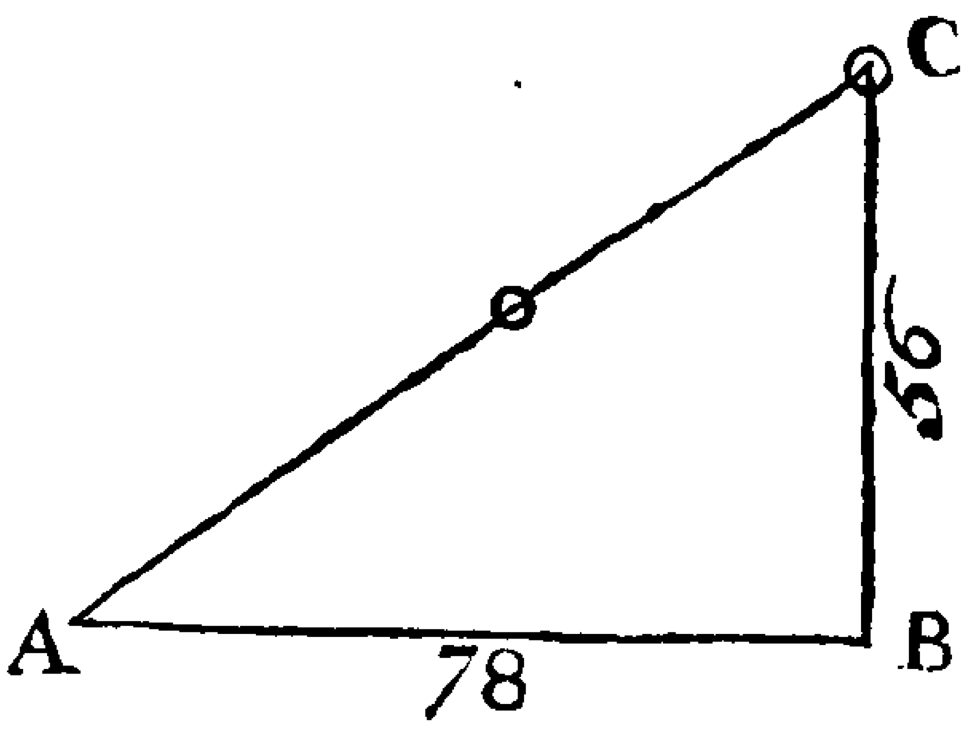
```
To Hypothenuſe  96
Add half longer Leg 39
                ---  :  Fixed Number  : :  Perpendicular
           Sum 135  ----    86    ----          56
                                                86
                                               ---
                                               336
                                              448
                                              ----
                                   135)4816.(35.67 + Angle A
                                       405
                                       ---
                                        766
                                        675
                                        ---
                                         910
                                         810
                                         ---
                                         1000
                                          945
                                          ---
                                           55
```

Anſwer, $\left\{ \begin{array}{l} \text{The Angle, } 35^\circ\ 41' \text{ nearly.} \\ \text{The Perpendicular, } 55.9 +\text{, or } 56. \end{array} \right.$

CASE IV.

The Two Legs given; to find the Hypothenuse and the Angles.

Given { Base AB = 78
Perpend. CB = 56 } to find { Hypoth. AC
and
Angles A & C.



(1st.) Find the Hypothenuse by Axiom II.

Perpendicular 56 78 Base
squar'd 56 78 squar'd

336 624
280 546

3136 6084 Square of Base
3136 Square of Perpendic.

Extract the Root 9220(96 The Hypothenuse
81

186)1120
1116

4

To Hypothenuse 96
Add half longer Leg 39

Sum 135 : Fixed Number : : Perpendicular
86 56
86

336
448

135)4816.(35.674+ Angle A
405

766
675

910
810

1000
945

550
540

10

Answer, { The Hypothenuse, 96.
The Angle, 35°.674, 35° 41′ nearly.

D NOTE.

Note. Thus all the Cafes of *Right Angled Triangles*, are eafily and readily anfwered: and by the fame Rules, and with the like Eafe may the *Oblique Angled Triangles* be anfwered, as will evidently appear in the following Cafes.

Of Oblique TRIANGLES.

In the Solution of an *Oblique Triangle*, it is neceffary, by this Method, to divide it into Two *Right Angled Triangles*, by means of a *Perpendicular*, which muft always fall upon the End of a given Side, and oppofite to a given Angle.

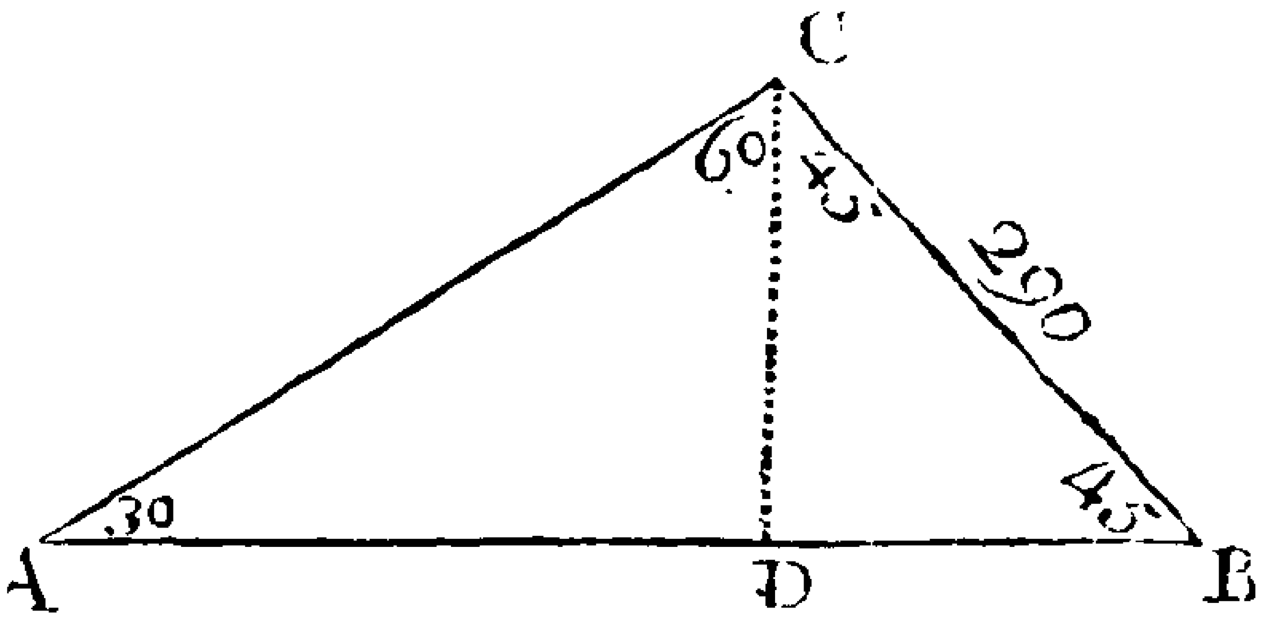
By this means the *Perpendicular* will fometimes fall *within*, and fometimes *without* the Triangle: When it falls within, it falls upon fome Part of the Bafe, or longeft Side; but when it falls without, it falls upon one of the fhorter Sides continued. In either Cafe, there are Two *Right Angled Triangles* made, and the Angles, or Sides fought, are found as if they were Parts of *a Right Angle*, by the foregoing Axioms; but it requires *Two* or *Three* Operations.

CASE

CASE I.

Two Angles, and a Side opposite to one of them, given; to find the other two Sides.

Given { Angle A 30, Angle B 45, Side BC 290 } to find { Side AC and Side AB.




```
  45        45
  45         3
 ----      ----
 225       135
180        300
 ----      ----
2025       435
   4
 ----
435)8100(18.6
    434  45.
    ----
    3750 63.6 Natural Radius
    3480
    ----
     2700
     2610
```

```
   30
   30
  ----
  900
    3
  ----
 2.700
 57.3
 ----
Natural Radius 60.0
```

(1st.) Find the Perpendicular CD.

```
 N. Rad.     Hypoth.     ∠ at B
As 63.6 ---- 290 ------- 45
              45
             ----
             1450
            1160
             ----
63.6)13050(205 Perpendicular CD
     1272
     ----
      3300
      3180
      ----
       120
```

(2d.) Find the Side AC.

```
∠ at A     Perp.CD    N. Rad.
As 30 ---- 205 ------ 60
            60
           ----
30)12300(410 The Side AC Hypothenuse
   120
   ---
    30
    30
    --
     0
```

(3d.) Find the Side AD.

```
       To Hypoth. AC 410
     Add Perpend. CD 205
                     ---
                 Sum 615
Multiply by Difference 205
                     ----
                     3075
                    12300
                    -----
Extract the Root 126075(354.9 AD
                  9
                 --
               65)360
                  325
                  ---
              704)3575
                  2819
                  ----
             7089)75900
                  63801
                  -----
                  12099
```

(4th.) Find the Side DB.

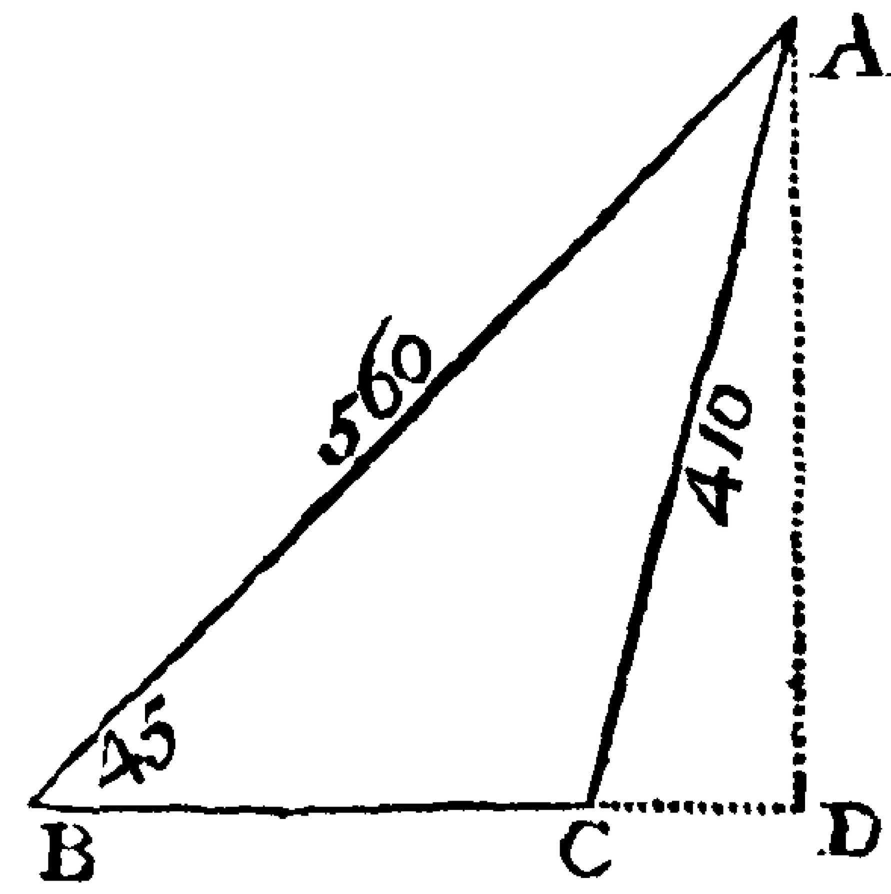
```
      To Hypoth CB 290
   Add Perpend. CD 205
                   ---
               Sum 495
Multiply by Difference 85
                   ----
                   2475
                  3960
                  -----
Extract the Root 42075(205 DB
                 4
                 --
              405)2075
                  2025
                  ----
                    50
```

Answer, { The Side AC 410. The Side AB 560. The Angle C 105.

CASE

CASE II.

Two Sides, and an Angle oppoſite to One of them, being given; to find the reſt.



Given	Side AB 560 Side AC 410 Angle B 45°	to find	Angle C and Side BC.

(1ſt.) Find the Side AD.

```
   Nat. Rad.      Hyp. BA       ∠B
As   63.6  ———    560   ———    45
                   45
                 ————
                  2800
                  2240
                 —————
          93.6)25200.0(396 AD
                 1908
                 ————
                  6120
                  5724
                  ————
                   3960
                   3816
                   ————
                    144
```

(2d.) Find CD.

```
        To Hypoth. CA  410
        Add Perpend. AD  396
                        ———
                   Sum  806
 Multiply by Difference  14
                        ———
                       3224
                       806
                      —————
Extract the Root  11284(106 CD
                  1
                 ———
              206)1284
                  1236
                  ————
                    48
```

(3d.) Find the Angle DAC.

```
  To Hyp. CA 410
Add ½ Perp. AD 198
              ——     Fix'd Num.    Side CD
          As 608 ——     86     ——    106
                                      86
                                     ———
                                     636
                                    848
                                    ———
                          608)9116(14.9+, or 15° ∠A
                               608
                               ———
                               3036
                               2432
                               ————
                               6040
                               5472
                               ————
                                568
```

(4th.) Find Side BD.

```
   To Hypoth BA  560
 Add Perpend. DA  396
                  ———
             Sum  956
Multiply by Differ. 164
                  ———
                 3824
                5736
               956
              ——————
Extract the Root 156784(395.9 + or 396 BD
               9..
               ——
           69)667
              621
              ———
          785)4685
              3925
              —————
         7909)75900
              71181
              —————
               4719
```

Then from BD = 396
Take —— CD = 106
——
Remains — BC = 290 Required.

Hence we find by Inſpection,
The Angle ACD = 75
The Angle ACB = 105
and The Angle BAC = 30° Required.

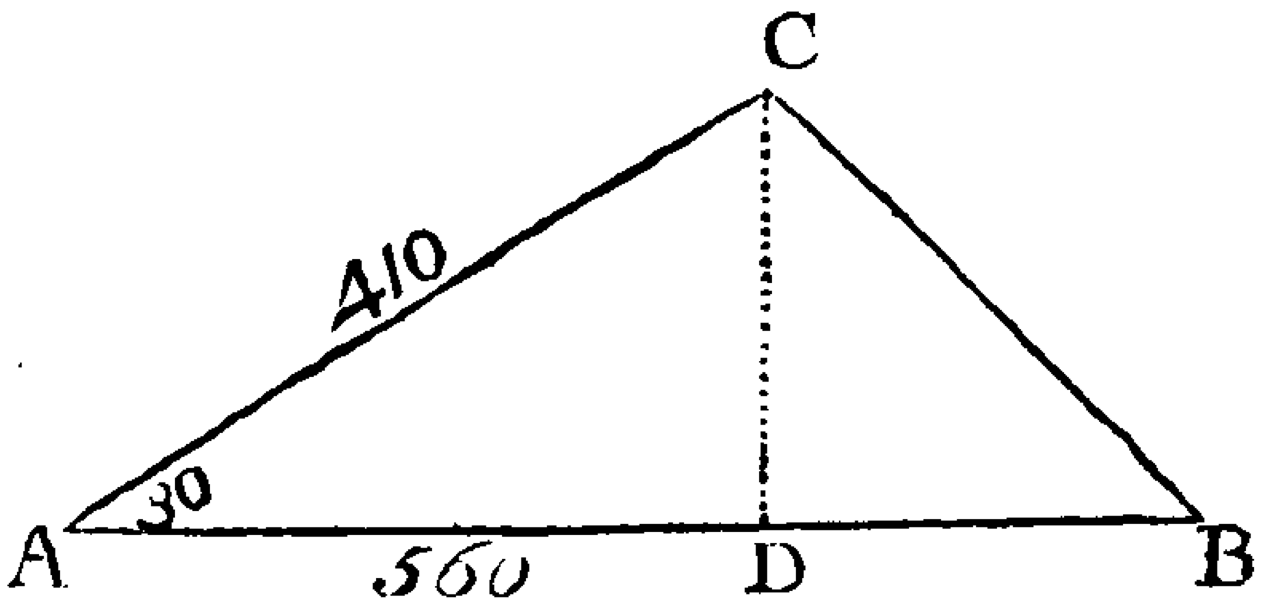
⁎ In this, and ſeveral of the following *Caſes* and *Problems*, the Operation for finding the *Natural Radius* is omitted, for Want of Room.

CASE

CASE III.

Two Sides, with the Angle comprehended by them, given; to find the rest.

Given { Side AC 410, Side AB 560, Angle A 30° } to find { Side BC and Angles B & C.



(1st.) Find the Perpendicular CD.

```
     N. Rad.    Hypoth. AC   ∠ at A
As   60  ———  410  ———  30
               30
             ———
       60)12300(205 Perpendicular CD
          120
          ———
            300
            300
            ———
              0
```

(2d.) Find the Part AD.

```
    To Hypoth. AC  410
  Add Perpend. CD  205
                   ———
              Sum  615
 Multiply by Differ. 205
                   ————
                   3075
                  12300
                  —————
Extract the Root 126075(355 Base AD
                 9....
                 —
              65)360
                 325
                 ————
             705)3575
                 3525
                 ————
                   50

    From AB = 560
    Take AD = 355
              ———
 Remains BD = 205
```

(3d.) Find the Side BC.

```
BD squar'd 205        205 CD squar'd
           205        205
           ———        ———
          1025       1025
          4100       4100
          —————      —————
To Square of DB 42025      42025
Add Square of CD 42025
                 —————
Extract the Root 84050(289.9 +, or 290 BC
                 4
                 —
              48)440
                 384
                 ————
             569)5650
                 5121
                 —————
            5789)52900
                 52101
                 —————
                   799
```

(4th.) Find the Angle B.

```
To  290   the Side BC
Add 102 5 half DB
    ———        Fix'd Num.   Side DC
As  392.5 ———  86     ———   205
                             86
                            ————
                            1230
                            1640
                            —————
                 3925)17630.0(44.9 +, or 45° ∠B
                      15700
                      —————
                       19300
                       15700
                       —————
                        36000
                        35325
                        —————
                          675
```

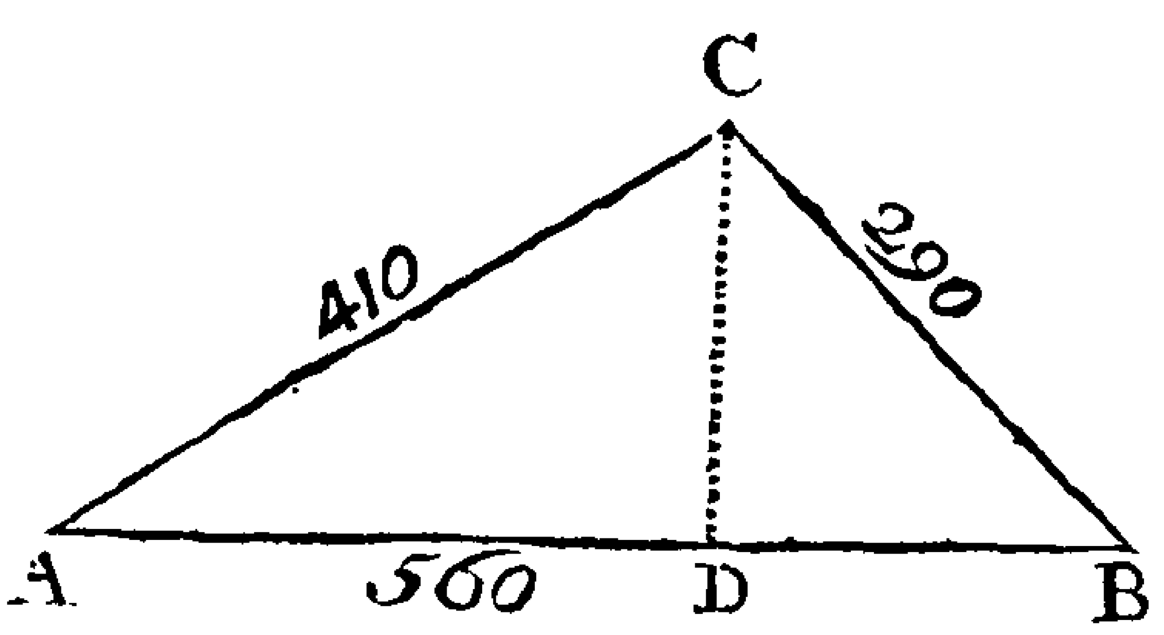
The Angle A = 30, added to Angle B = 45, and then subtracted from 180, leaves 105 for Angle C.

Answer, { The Side BC 290
The Angle B 45°
The Angle C 105°.

CASE IV.

The Three Sides given; to find the three Angles.

In all Triangles, as the Base or greater Side is to the Sum of the other Two Sides; so is the Difference of the Sides to the Difference of the Segments of the Base; which Half Difference, added to Half the Base, the Sum will be the *Greater Segment*, upon which the Perpendicular falls: But if subtracted from half the Base, the Remainder will be the *Less Segment*: So will the *Oblique Triangle* be reduced to two Right Angled ones, and may be answered after the same Manner as before.



Given		to find	
	Side AB 560		Angle A
	Side AC 410		Angle B
	Side BC 290		Angle C.

(1st.) Find the Segments of the Base.

As 560 —— 700 —— 120
120
14000
770
560)84000(150 Difference
560
2800
2800
0

To $\frac{1}{2}$ Base 280
Add $\frac{1}{2}$ Diff. 75
Gr. Seg. AD 355

From $\frac{1}{2}$ Base 280
Subtract $\frac{1}{2}$ Diff. 75
Less Seg. DB 205

(2d.) Find the Perpendicular CD.

To Hypoth. CA 410
Add Gr. Seg. AD 355
Sum 765
Multiply by Difference 55
3825
3825
Extract the Root 42075(205 CD
4
405)2075
2025
50

(3d.) Find the Angle at A.

To 410 Side AC
Add 177 half AD
Fix'd Num. Side DC
As 587 —— 86 —— 205
86
1230
1640
587)17630(30 ∠ A
1761
20

(4th.) Find the Angle at B.

To 290 Side BC
Add 102 half DB
Fix'd Num. Side DC
As 392 —— 86 —— 205
86
1230
1640
392)17630(44.9 +, or 45° ∠ B
1568
1950
1568
3820
3528
292

Then the Angle A = 30°, added to B = 45°, and subtracted from 180°, leaves 105° for the Angle C, which were the Angles required.

The

The USE of TRIGONOMETRY

Exhibited in the Solutions *of a Number of interesting* Problems; *many of which every Day occur; are of the greatest Utility in the* Army, Navy, *&c. and cannot be answer'd without it.*

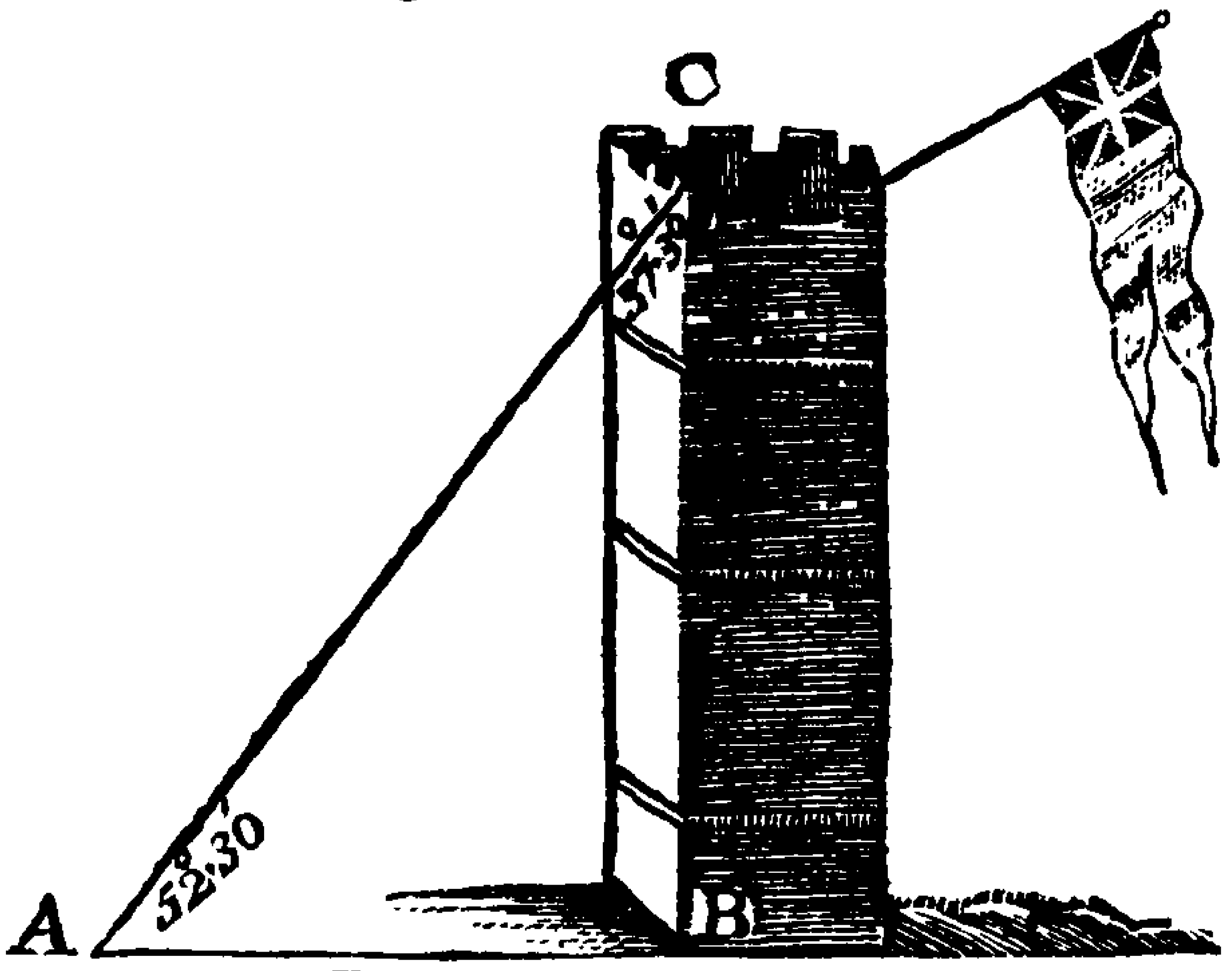
PROBLEM I.

To take the Height of any *accessible Object* at one Station.

First, with a *Quadrant*, by looking through the Sights to the Top of the Tower, find the Quantity of the Angle A, which suppose 52° 30′; then measure the Distance AB, which suppose to be 85 Feet; from hence you may proceed to find the *Height*, by Case I. of Right Angled Triangles.

```
        37 5
        37 5
        ----
        1875
       2625
      1125
      -------
     1406.25
           3
     -------
1000)4.218.75
     57 3
     --------
     61.51875 Natural Radius
```

But 61.52 is exact enough in Practice.



(1st.) Find the Hypothenuse AC.

```
   Angle C  :  Base  ::  Nat. Rad.
As 37.5  ——  85  ——  61.52
                        85
                     -----
                     30760
                    49216
                    ------
          37.5)5229.20.(139.44 + Hyp.
               375
               ---
               1479
               1125
               ----
                3542
                3375
                ----
                 1670
                 1500
                 ----
                  1700
                  1500
                  ----
                   200
```

(2d.) Find the Perpendicular BC.

```
   To Hypothenuse  139.44
         Add Base   85.
                   ------
              Sum  224.44
Multiply by Difference  54.44
                   ------
                    89776
                   89776
                  89776
                 112220
                 ------
Extract the Root 12218.5136(110.537 Perpendic.
                 1........
                 -
              21)22
                 21
                 --
            2205)11851
                  11025
                  -----
           22103)82636
                  66309
                  -----
          221067)1632700
                  1547469
                  -------
                    85231
```

Answer, 110.537 +, or 110 Feet, and above ½ : The Height Required.

NOTE; That in this, and all such Cases, you must add the *Height* of your *Eye*, or *Instrument* to the *Altitude* before found.

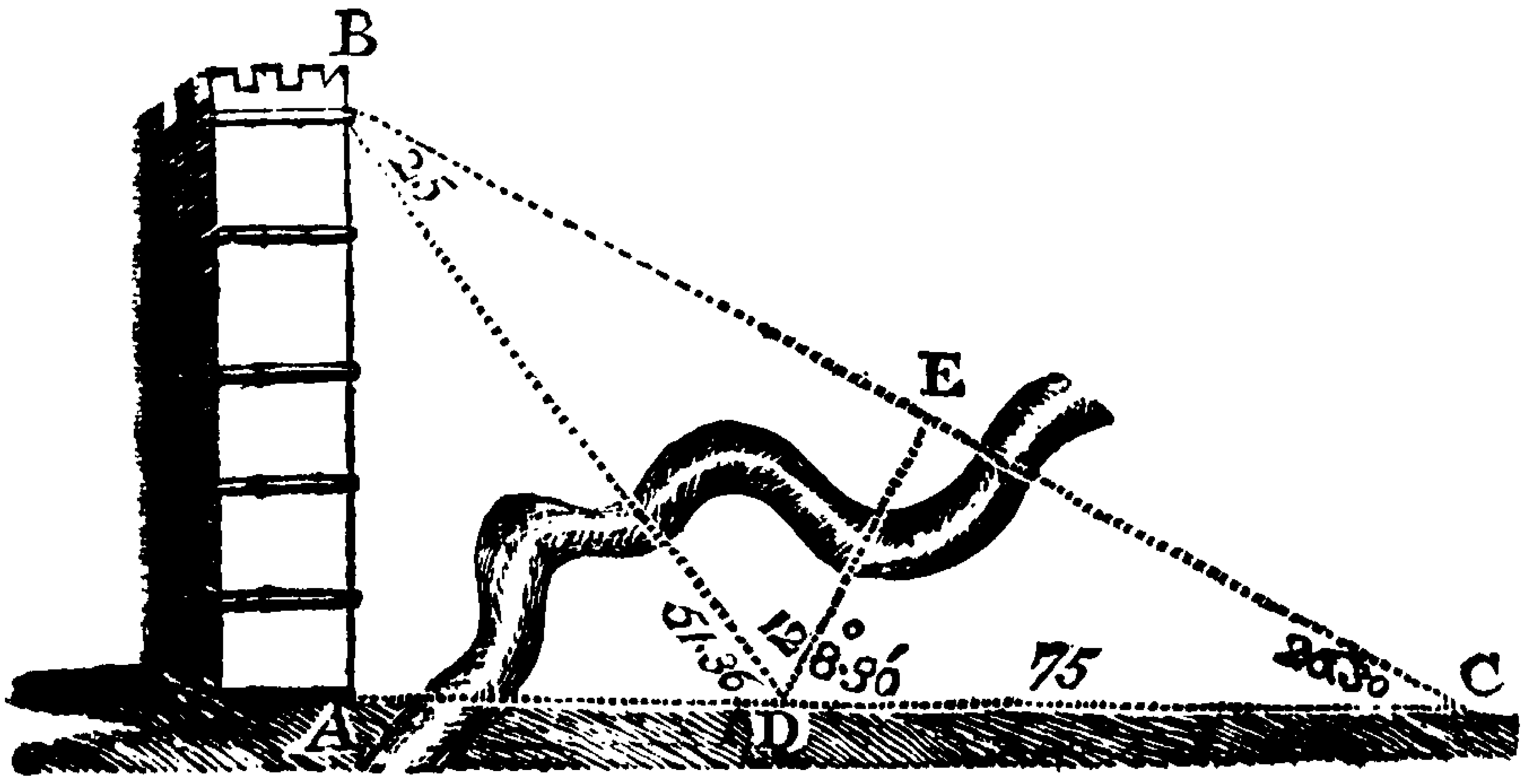
PROBLEM

PROBLEM II.

To meaſure an *Inacceſſible Altitude.*

Let AB, in the following Figure, be a *Church, Tower,* or *Fort,* whoſe Height is required; but by Reaſon of a River, or ſome other *Obſtacle,* it is *inacceſſible*; that is, you cannot come to the Foot of it, by Reaſon of the *Water,* &c.

Firſt, with a *Quadrant,* take the Angle of *Altitude* at C, which ſuppoſe 26° 30′. Then meaſure in a Right Line towards the *Tower* to D, any Diſtance, ſuppoſe 75 Feet, and at D obſerve again the Angle of *Altitude,* which let be 51° 30′.



Then; the two Viſual Lines CB and DB, with the Diſtance DC, make the Oblique Triangle CBD, in which are given—All the Angles and Side CD. The Angles BCD being 26° 30′, and the Complement of ADB 51° 30′ to 180, is the Obtuſe Angle BDC 128° 30′. Conſequently, the third Angle CBD, at the Top, is = 25°.

(1ſt.) Find the Perpendicular DE in Triangle DBC.

Nat. Rad. : Op. Side DC :: Ang. C : Perp. DE
As 59·4 — 75 — 26·5 — 33·46

(2d.) Find the Viſual Line BD in Triangle BDE.

Ang. B : Op. Side DE :: N. Rad. : Side BD
As 25 — 33·46 — 59·17 — 79·19

(3d.) Find the Height AB in the Right Angled Triangle ABD.

Nat. Rad. : Op. Side BD :: Ang. D : Height
As 65·7 — 79·19 — 51·5 — 62 AB

(4th.) Find the Diſtance AD in the Triangle ABD.

N. Rad : Op. Side BD :: Ang. ABD : Dif. AD
As 61·7 — 79·19 — 38·5 — 49·41

Anſwer, { 62 Feet the Height.
49·41, or 49½ Feet the Diſtance from the ſecond Station.

NOTE. The Line BD is the Length of a *Scaling Ladder,* which would reach from the *Station* at D over the *Foſs* or *Ditch,* to the *Top* of the *Tower* at B.

PROBLEM

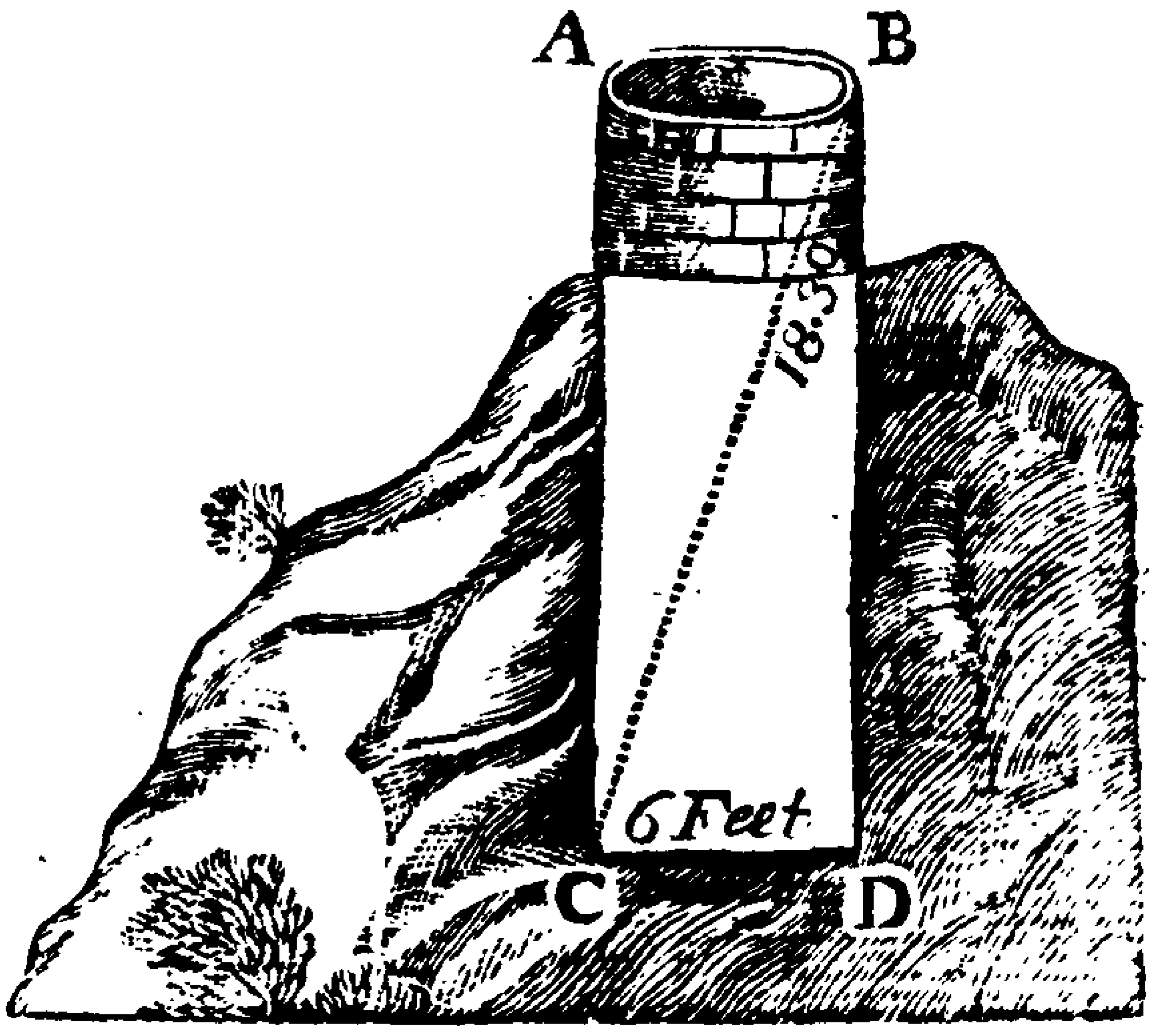
PROBLEM III.

To meaſure the *Depth* of a *Well*, or the *Height* of an *Object* from the *Top* of it.

Firſt, look through the Sights of the *Quadrant* to the Bottom of the oppoſite Side the Well at C, ſo you will have the *Angle* CBD; next, take the *Breadth* AB at the Top, which is equal to CD at the Bottom: Then, by Caſe I. of Right Angle Triangles, you may eaſily find the *Depth* BD required.

Suppoſe the Angle at B, by Obſervation, to be 18° 30′, and the Breadth at the Top 6 Feet; what's the *Depth*.

```
   18.5
   185
   ----
    925
  1480
  185
  -----
  342.25
       3
  -------
 1.026.75
57.3
---------
Natural Rad. 58.32675 but 58.3 is enough.
```



(1ſt.) Find the Hypothenuſe BC.

```
   ∠B   Op. Side   N. Rad.
As 18.5 — 6 —— 58.3
                  6
               ----
        18.5)349.8.(18.9 + the Line BC
             185
             ----
             1648
             1480
             ----
              1680
              1665
              ----
                15
```

(2d.) Find the Depth BD.

```
To Hypoth. BC 18.9
  Add Side CD  6.
              ----
          Sum 24.9
Multiply by Differ. 12.9
              ----
              2241
              498
             249
             ------
Extract the Root 321.21(17.89 the Depth AB
                 1
                ---
              27)221
                 189
                 ----
             348)3221
                 2784
                 -----
            3569)43700
                 32121
                 -----
                 11579
```

Anſwer, { 17.89+, or 18 Feet, the Depth required.

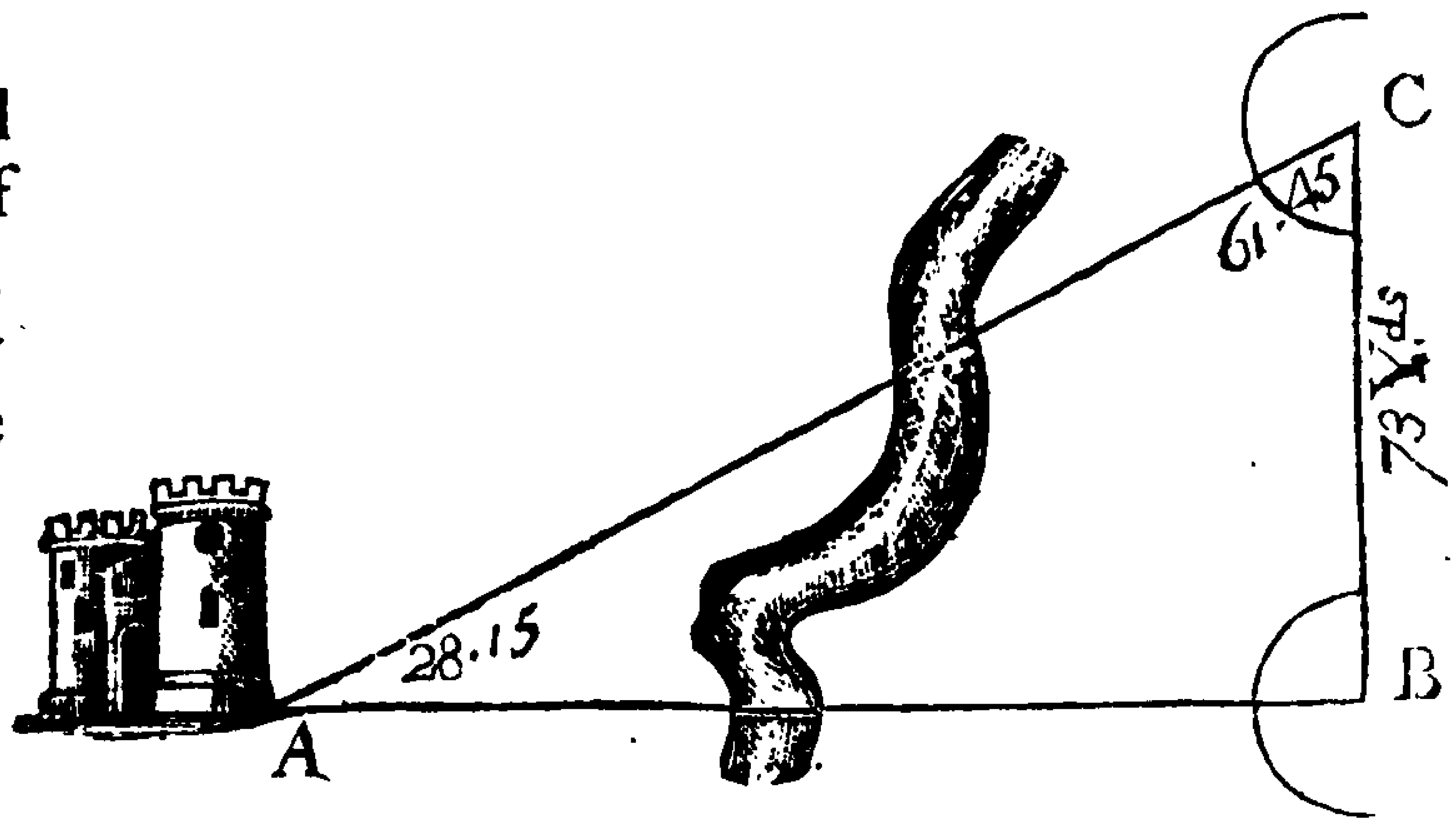
PROBLEM IV.

To measure the *Distance* of any *Object*.

Suppose yourself standing at B, and a great Way off, as at A, you see a *Fort* or *Castle*, &c. or any other *Object*, whose *Distance* you would find from the Place where you now stand.

First, a *Theodolite*, or *Semicircle*, being placed at B, lay the *Index*, with its Sights, on the *Diameter*, where the Degrees begin, and through them view the *Castle*, &c. at A. The Instrument remaining fix'd in this Position, move the *Index* to 90 Degrees, (that being a Right Angle) and view some Mark at a Distance, (the farther off the better) as at C.—Next measuring the Distance from B to C, which suppose 73 Yards, remove your Instrument, and set it up at C. Then, with the Index laid upon the Beginning of the Degrees, as before, turn the Instrument about, till you can see your *first Station* at B, where fasten it; then turn the Index till you can see the Object A, and observe what Degrees are cut, as suppose 61° 45′, which is the Quantity of the Angle where you stand; whose Complement to 90° is the Angle A.

Now, here are given all the Angles, and one Side of a *Right Angled* Triangle, to find either of the other Sides, which will be the *Distance* required.




```
    28.25
    28.25
   ------
    14125
    5650
   22600
   5650
   ------
  798.0625
         3
  ---------
 2.394.1875
 57.3
 ----
 59.69 or 59.7 Natural Radius
```

(1st.) Find the Distance from C.

Ang. A : Op. Side BC :: N. Rad. : Dist. CA
As 28.25 — 73 — 59.7 — 154.2

yards
Ans. { 135.8+, or 136 Dist. from B.
       { 154.2, or 154¼ Dist. from C.

(2d.) Find the Distance from B.

```
To Hypoth. AC   154.2
Add Side BC      73
                -----
Sum             227.2
Multiply by Differ. 81.2
                -----
                 4544
                2272
               18176
               ------
Extract the Root 18448.64(135.8+, Dist. from B
                 1
                 -
               23)84
                  69
                  --
              265)1548
                  1325
                  ----
             2708)21364
                  21664
                  -----
                    700
```

PROBLEM V.

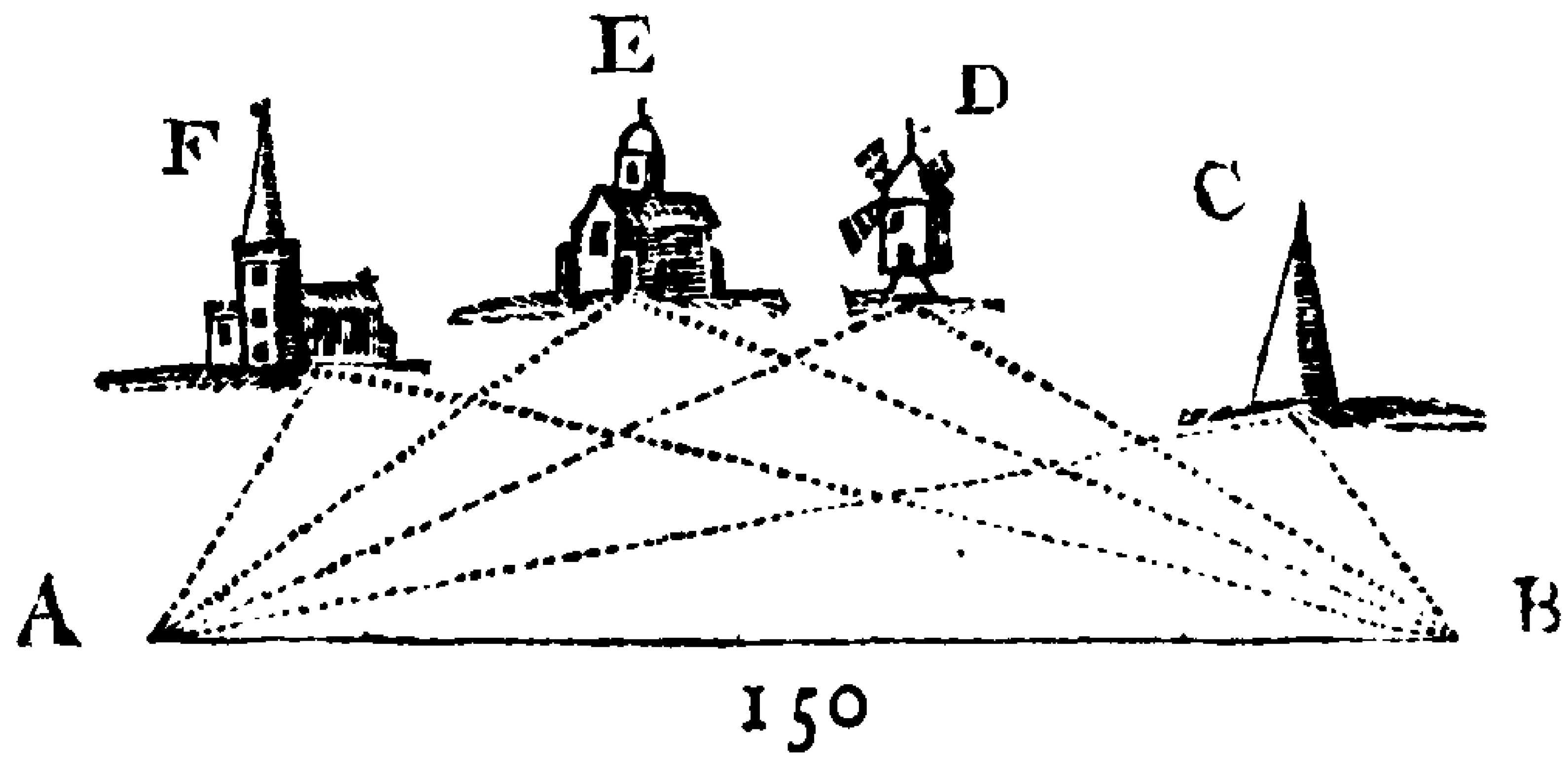
To take the *Distances* of several *inaccessible Objects*, as *Forts*, *Churches*, in a *Town*, or *Squadron of Ships* at *Sea*, and to delineate them upon *Paper*.

First, make choice of two Places, from either of which you may conveniently see all the *Objects*; which two Places let be A and B in the following Figure.—This being done, set up your Instrument at A, laying the *Index* on the *Diameter*, and turn the whole Instrument about, till, through the Sights, you see your second Station B. Then, fixing the Instrument, direct your Sights to the several *Objects*, C, D, E, and F; noting down the Degrees cut at each Observation, which suppose to be as in the Table.

Then, remove the Instrument to B, laying the *Index* on the *Diameter*, and turn it about, till, through the Sights, you see your former *Station* at A; then direct your Sights to every one of the Objects at C, D, *&c.* setting down the Degrees at each Observation, as in the Table. Also measure the *Stationary Distance*, and set that down.

	C	D	E	F
1st Station	11	23	36	59
2d Station	51	31	22	13
Stationary Distance 150 Yards				

First, upon a Piece of Paper draw the Line AB; and from a *Scale of equal Parts*, take off, with your Dividers, the *Stationary Distance* = 150, and set it from A to B, so will A represent your *first Station*, and B the *second*. Then lay the *Center* of the *Protractor* upon the Point A, with its Diameter upon



the Line AB; keeping it fast, make Marks by the Edge at 11, 23, 36, 59, and draw Lines from the Point A through each of those Marks. Then upon B place the *Center* of your *Protractor*, its Diameter lying upon the Line AB; make Marks by the Side at 13, 22, 31, 51. Then draw Lines from the Point B through each of these Marks, and where the Lines cut the former *correspondent Lines* there will be found the Places repesenting these *Objects*. Then any of these Lines being taken in a Pair of Dividers, and applied to the Scale you laid your *Stationary Distance* down by, will give you their *Distances*, either from your *Stations* or from one another.

The *Distance* of any of these *Objects* from *either Station*, &c. may be found by *Calculation*; *one Side* and the *Angles* being given: But I shall omit that, on purpose to *exercise* the *Learner*'s *Genius*, and proceed.

PROBLEM

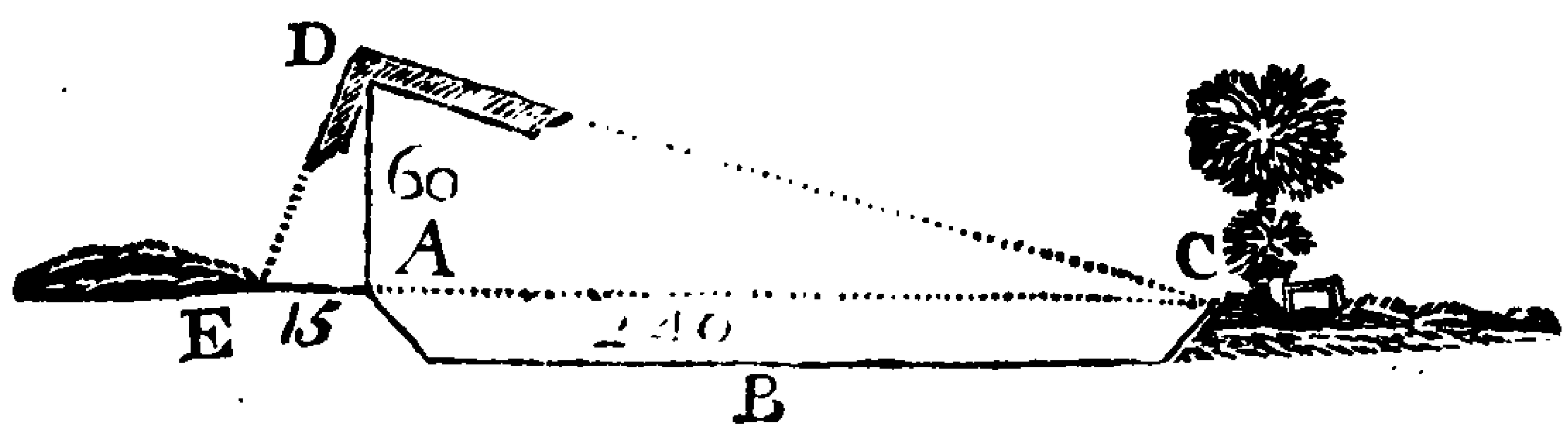
PROBLEM VI.

To take the *Distance*, upon level Ground, of any *inaccessible Tree*, *Fort*, &c. or *Breadth* of a *River*, by a *Common Square*.

Suppose there is a River, as A B C, whose *Breadth* you want to know. —First, upon the Bank, at A, set up a Stick, AD, which suppose to be 5 Feet, or 60 Inches high; then fixing your *Square* on the Top, at D, look by the Side of it till you see the Edge of the opposite Shore C, and fasten it, as it may not go from that Position. This done, extend a Thread from D, by the other Side of the *Square* till it touch the Ground at E. Then measure the Distance EA, which suppose 15 Inches, (or 1 Foot 3 Inches) and you may find AC (by Reason of similar Triangles) thus.

	Dist. EA	:	Side DA	: :	Side DA	:	Dist. AC
As	15	——	60	——	60	——	240 Inches,

which, reduc'd to Feet, give 20 for the Breadth of the *River* sought.



NOTE. There are various Ways of taking *Heights* and *Distances*; but the *best* is to take the Angles for *Heights* by a *Quadrant*; and the Angles for *Distances* by a *Semicircle* or *Theodolite*; and calculate by the foregoing *Axioms*. In all *Heights* the *Triangle* stands *upright*; but in *Distances*, it is suppos'd to lie *flat* or *horizontal*.

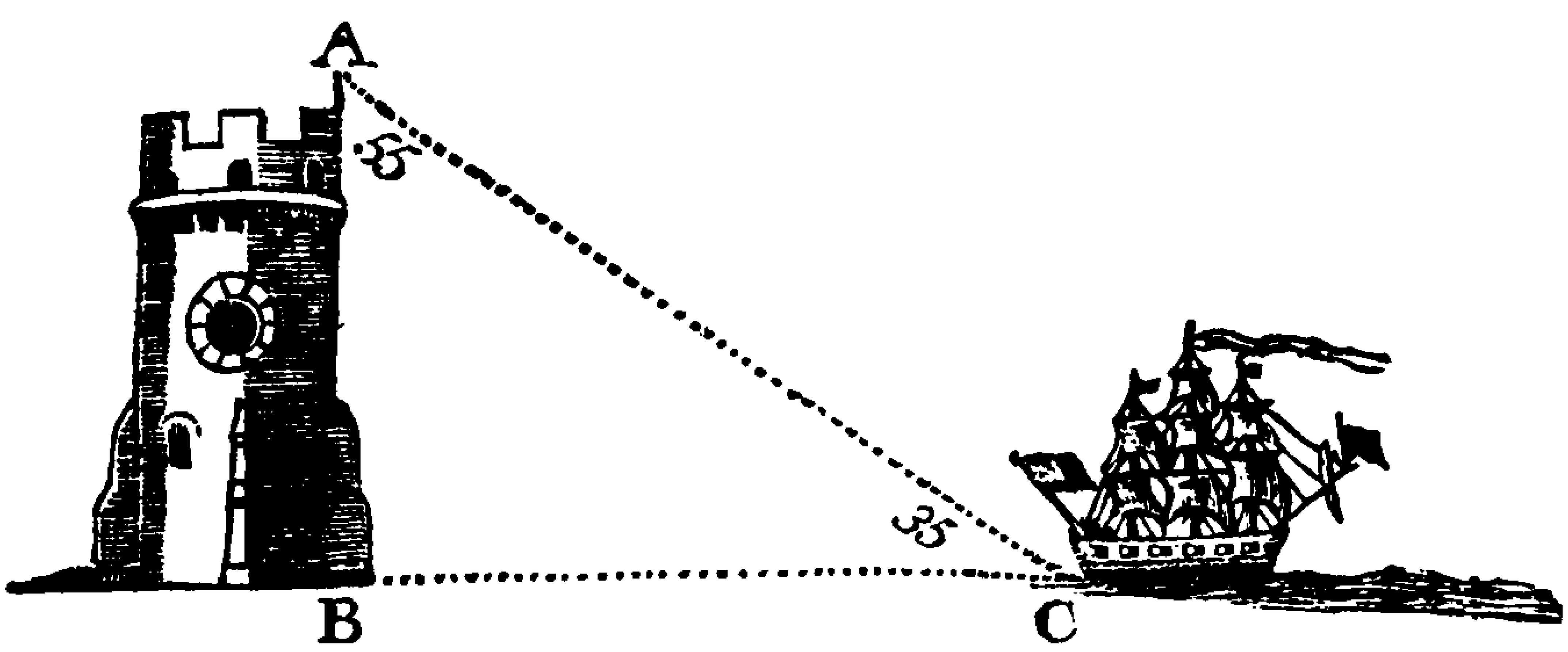
☞ I shall here shew the Learner, how to take the *Breadth* of a *River*, or a *small Distance*, without any *Instrument whatever*; which is thus. Standing upon the Bank, bring down the Edge of your *Hat*, till it appears to touch the opposite Side, then steady your Head by laying your Hand under your Chin, and turn yourself towards some *level Ground*, observing where the Edge of your *Hat* glances upon it; for then, the *Distance* from you to that *Place*, is *equal* to the *Breadth* of the *River*, or *Distance* required.

PROBLEM VII.

To find, from the Top of a *Fort*, or *Tower*, how far any *Tree*, *Ship*, &c. is from you.

Let A be the Top of a *Tower* or *Castle* standing by the *Sea-side*; and let C be a *Ship* at Sea, or lying at Anchor, and you would know how far that Ship is off the *Castle-wall*.

With your *Quadrant* or *Semicircle*, direct your Sights from the Top of the *Tower* to the Place where the *Ship* is, and take the Angle, which we will suppose to be 55 Degrees. Then the *Castle-wall* being known before to be 143 Feet high, you may easily find the *Distance* of the *Ship* from the *Wall* in this Manner.



(1st.) Find the Side AC.

Ang. C	: Height Wall	:: Nat. Rad.	: AC
As 35	— 143	— 61	— 249.2

(2d.) Find the Base BC.

N. Rad.	: AC	:: Ang. A	: Dist BC
As 67	— 249.2	— 55	— 204.56

By this Method you may easily discover if a *Fleet* of *Ships*, or one *single Ship*, at Sea, makes towards you or not. For having observ'd from the Top of the *Fort* the Angle from thence to the *Ship*, and noted it down, rest a little Time, and observe again: Then, if the Angle be *bigger* than before, the Ship is *departing* from you; but if *less* she is *making towards* you.

PROBLEM VIII.

To take the *Perpendicular Height* of a *Hill* or *Mountain*, and alſo the *Horizontal Line*, or *Baſe*, on which it ſtands.

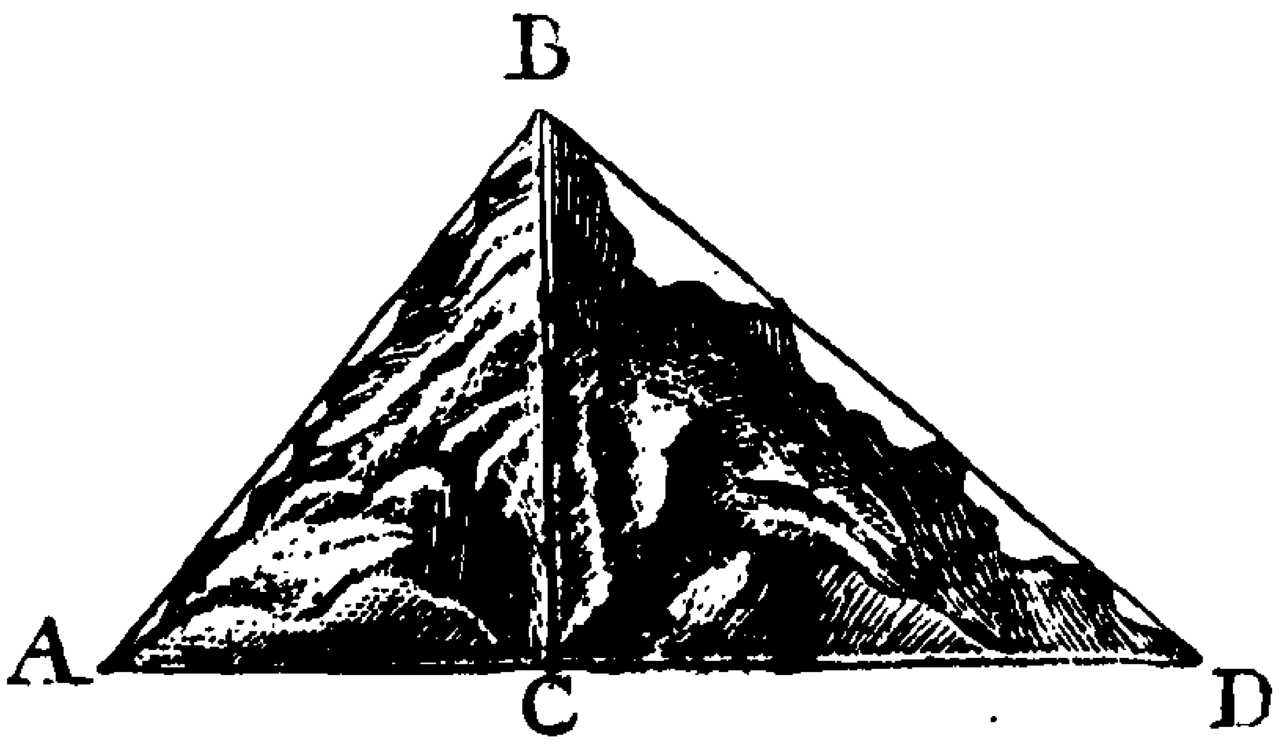
Let ABD be the *Hill*.—Firſt, ſet up a Mark on the Top at B, equal to the Height of the *Quadrant* or Inſtrument that is uſed at the Bottom, from whence you intend to make your Obſervation. Then by looking through the Sights to B take the Quantity of the Angle at A, which we will ſuppoſe to be 50°. Next meaſure the Hill from A to B, which let be 546 Feet. This being done, you may eaſily find the *Perpendicular* BC, or Part of the *Baſe* AC, by Caſe II. of Right Angles.

For the Perpendicular BC.

N.Rad.	: Side AB	:: Ang. A	: Perp. BC
As 65.2	— 546	— 50	— 418.7

For the Side AC.

N. Rad.	: Side AB	:: Ang B	: Side AC
As 62.2	— 546	— 40	— 351.1



Now, as the Hill *deſcends*, you may go on the oppoſite Side, and make the like Obſervations, viz. ſet up the Inſtrument at D, and take the Angle D, 40°, and meaſure the Side DB, 651 Feet, then you may find the Side CD in the ſame manner you did AC. Thus,

Nat. Rad.	: Side DB	:: Ang. B	: Side CD
As 65.23	— 651	— 50	— 499.

If to the Part AC = 531.1 Feet, be added the Part CD = 499 Feet, the Sum 850.1 Feet will be the whole Length of the *Horizontal Line* AD requir'd.

The *Perpendicular Height* DC is = 418.7 as above.

PROBLEM

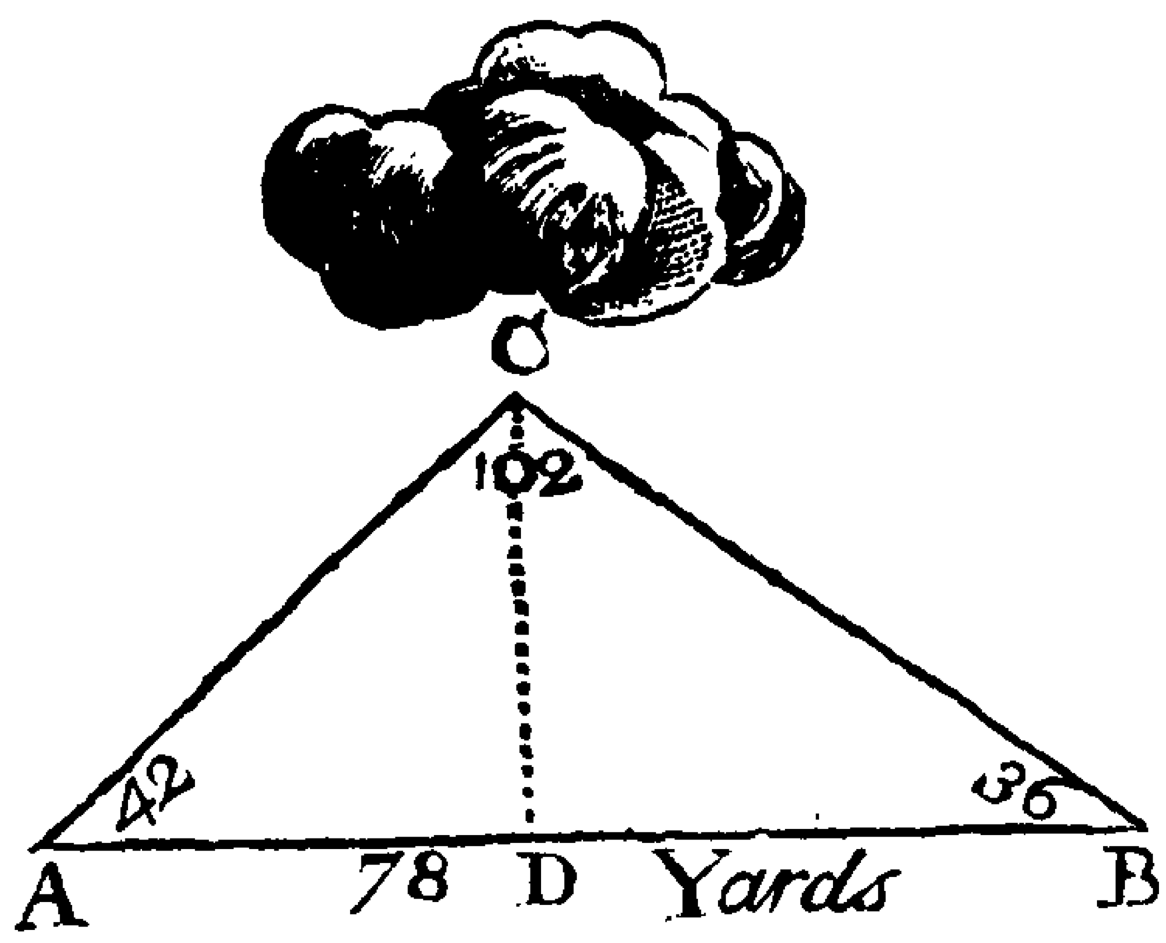
PROBLEM IX.

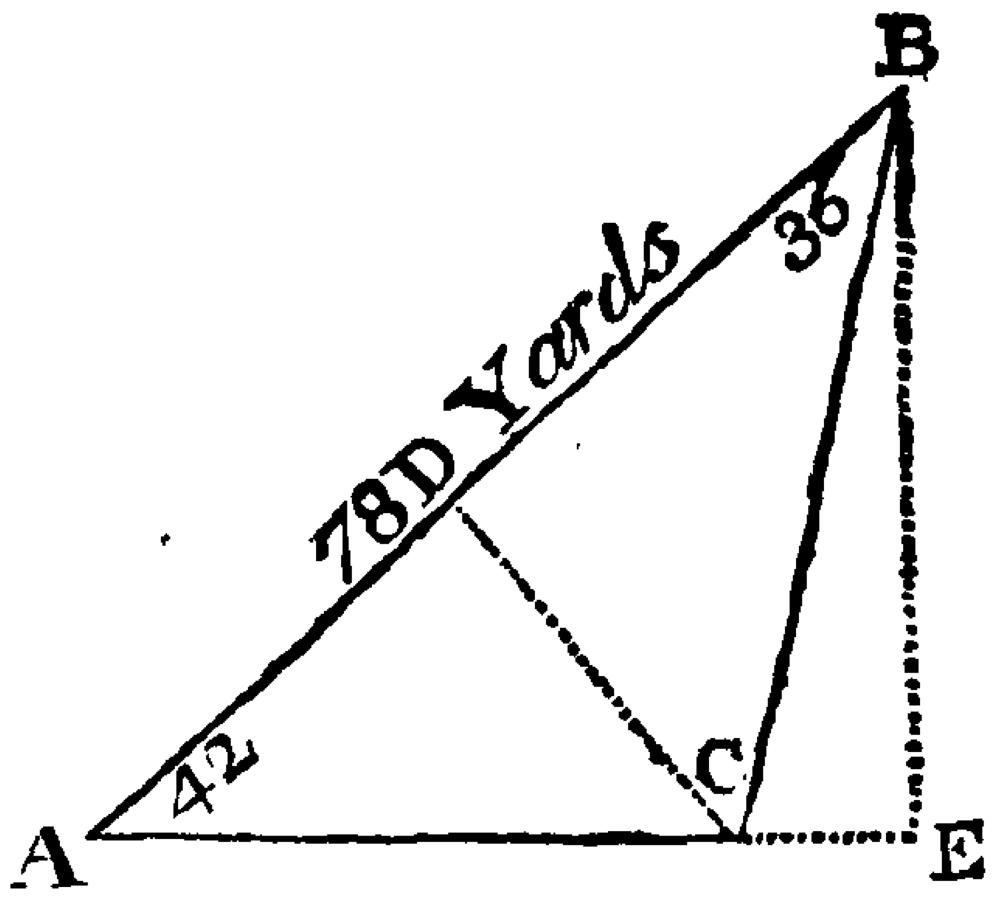
To take the *Height* and *Distance* of a *Cloud.*

Suppose it was requir'd to find the Height of the *Cloud* C.

Let a Person standing at A, look through the *Quadrant* to the *Cloud* at C, so will the Thread cut the Angle at A. At the same Time let another Person, making the like Observation at B, take the Angle B. Then measure the *Distance* between the *two Stations*. By this means you will have *one Side* and *all* the *Angles* of an *Oblique Angle Triangle* given, from whence you may easily obtain the rest, and particularly the *Perpendicular* CD, which will be the *Height* of the *Cloud* requir'd.

EXAMPLE. Suppose the Angle at A, by Observation, be 42°, the Angle B 36°, and the Distance AB 78 Yards: I demand the *Height* of the *Cloud.*





(1st.) Find the Perpendicular BE in Triangle ABE *.

N. Rad. : Op. Side AB : : Ang. A : Perp. BE
As 62.6 —— 78 —— 42 —— 52.1

(2d.) Find the Hypothenuse CB in Triangle CBE.

Ang. BCE : Op. Side BE : : N. Rad. : Hyp. BC
As 78 —— 521 —— 79 —— 53.2

(3d.) Find the Perpendicular CD in Triangle CDB.

N. Rad. : Op. Side BC : : Ang DBA : Perp. CD
As 61.1 — 53.2 — 36 — 31.3

Answer, 31.3 Yards, the Height requir'd.

* The Figure on the *Right Hand* is only that on the *Left* set in a different Position, to shew in a more ~~natural or~~ easy Manner, how the Perpendicular falls from the End of the given Side AB, upon the Side AC produc'd to E.

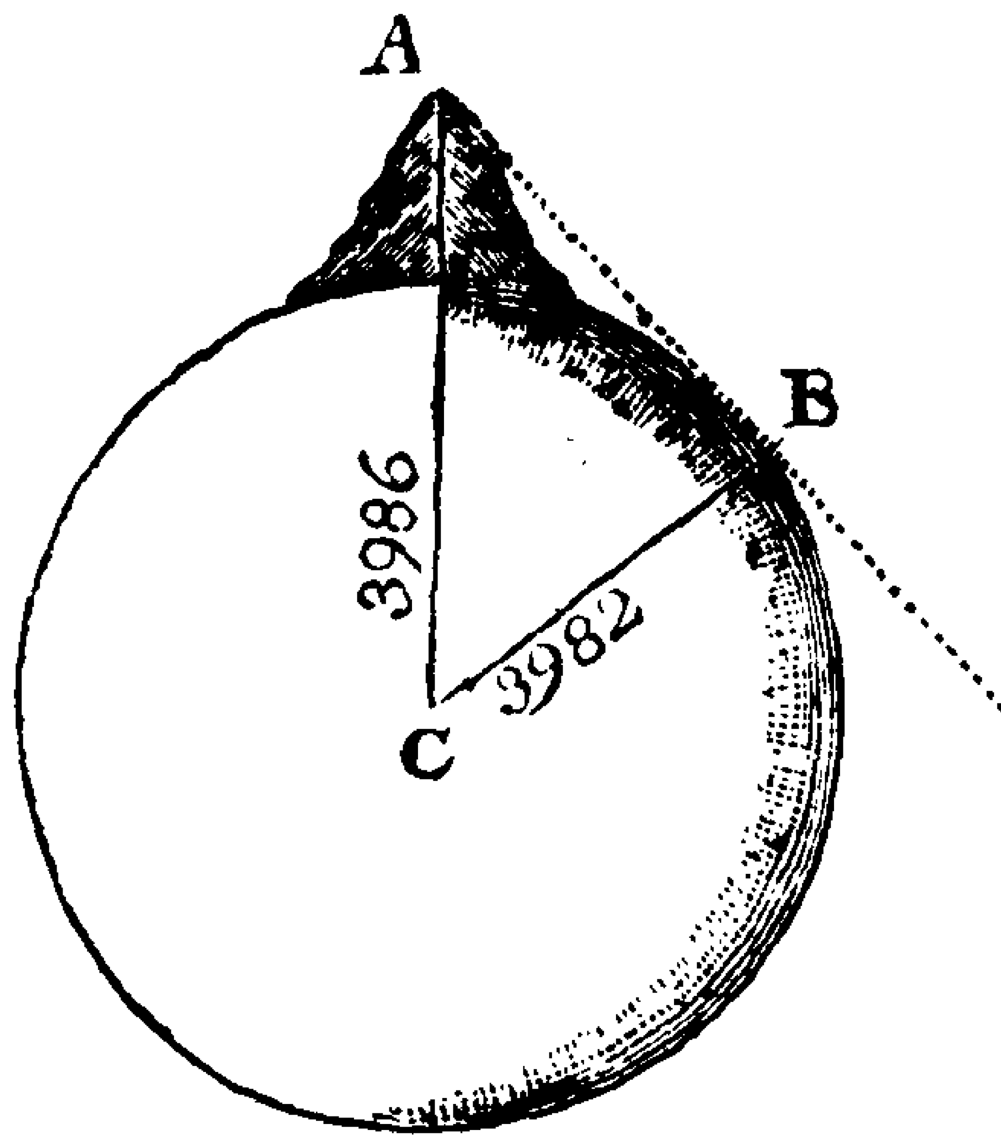
PROBLEM

PROBLEM X.

To find how far a *Hill* of any given *Height* can be ſeen at *Sea*, or upon *level Ground.*

How far, for Inſtance, can the *Pike* of *Teneriff* be ſeen at *Sea*, whoſe Height is about *four Miles*.

☞ The *Circumference* of the *Earth* is ſuppos'd by *Mathematicians* to be divided into 360 equal Parts, called *Degrees*; and our countryman, Mr. *Norwood*, has found, by meaſuring from the *Tower* of *London* to the Middle of the City of *York*, in the Year 1635, that one of thoſe Degrees, upon the Earth's Surface, contains $69\frac{1}{2}$ Miles; according to which Meaſure, we find the Earth's *Circumference* to be 25020 Miles—its *Diameter* 7964—and its *Semidiameter* 3982.



Then in the Triangle ABC, Right Angled at B, we have the Side CB = the Earth's Semidiameter 3982.—Alſo the Line AC = the Semidiameter and Height of the Mountain together = 3986.—To find AB, the Diſtance from the Hill to the viſible Horizon.

To Hypoth. AC 3986
Add Leg BC 3982

Sum 7968
Multiply by Differ. 4

Extract the Root 31872.00(178.5 The Diſtance the Mountain can be ſeen.
1

27)218
189

348)2972
2784

3565)18800
17825

975

This *Mountain* can be ſeen 178.5 Miles at *Sea*.

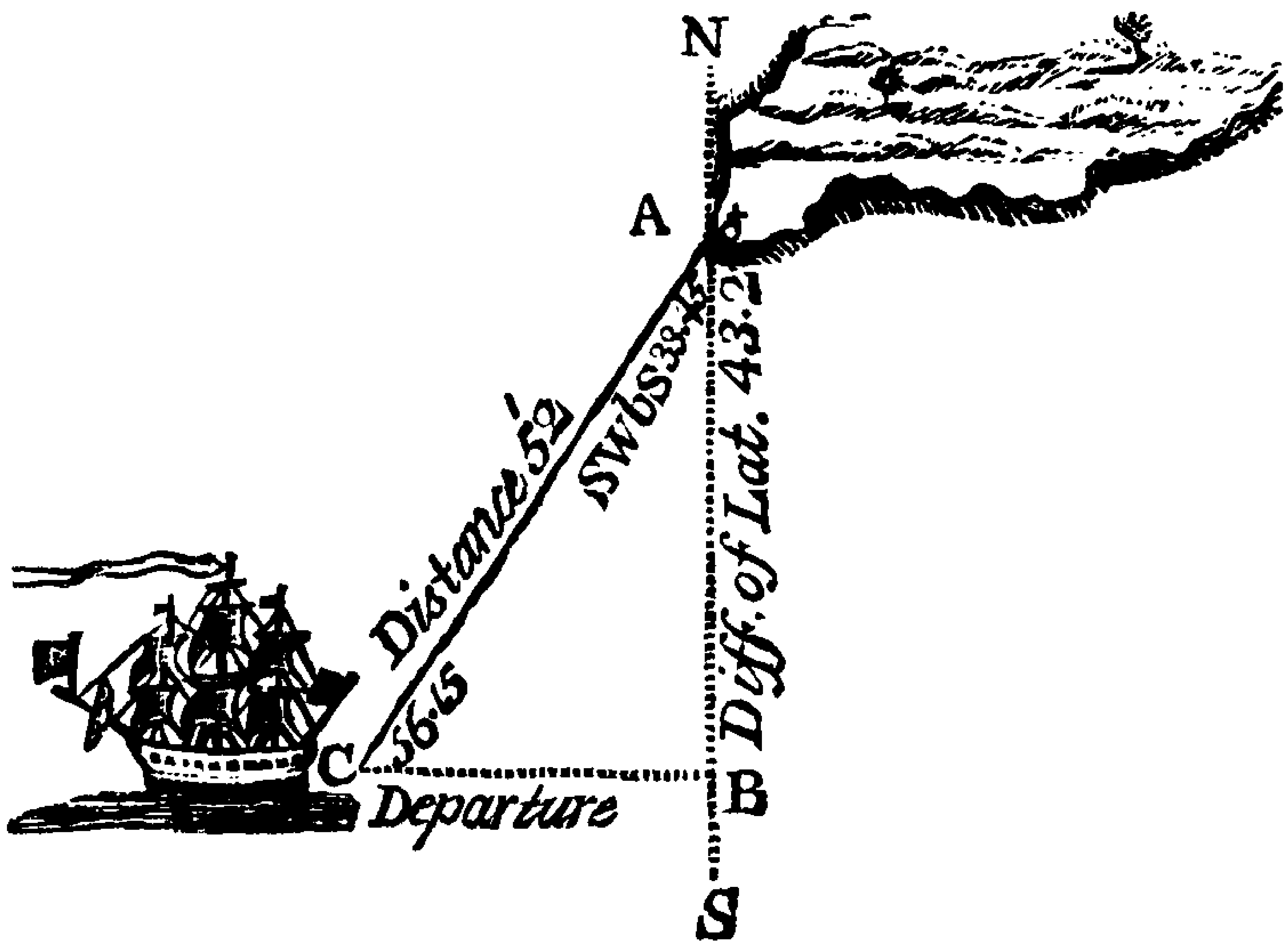
PROBLEM

PROBLEM XI.

The *Distance run* at Sea, and the *Course*, given; to find the *Difference* of *Latitude* and *Departure* from the *Meridian*.

Suppose a *Ship* from A, in the *Latitude* of 50° North, sails away, SW by S. 52 Miles, to C: I demand the *Latitude* she is in, and also her *Departure* from the *Meridian*.

33.75
33.75
———
16875
23625
10125
10125
———
1139.0625
3
———
3.417.1875
57.3
——
Nat. Rad. 60.7 is enough.



(1st.) For the Departure.

N. Rad. Hyp. AC ∠ A
As 60.7 — 52 — 33.75
52
———
6750
16875
———
60.7)1755 00(28.9 The Departure CB
1214
———
5410
4856
———
5540
5463
———
77

(2d.) For the Diff. of Latitude AB.

To Hypoth. AC 52
Add Side BC 28 9
———
Sum 80.9
Multiply by Differ. 23.1
———
809
2427
1618
———
Extract the Root 1868.79(43.2 Diff. of Latitude
16
———
83)268
249
———
862).979
1724
———
255

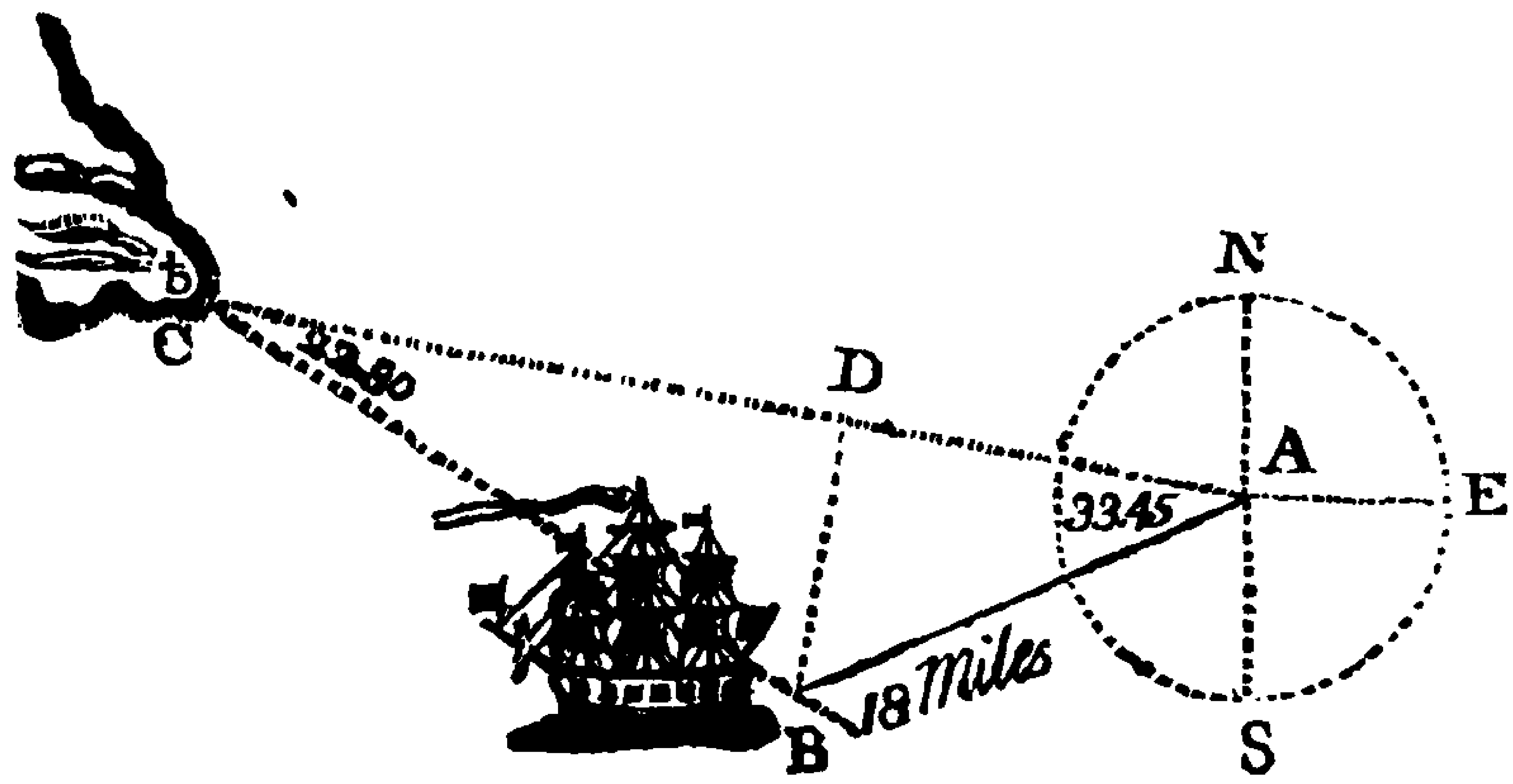
Latitude departed from	50°	North
Difference of Latitude	0	43.2
Latitude the Ship is in	49	16.8

NOTE. That in all Cases of *Sailing*, we suppose the *Top* of the Book *North*, and *Bottom South*; the *Right Hand East*, *Left Hand West*.—The *Distance run* is the *Hypothenuse*; the *Difference of Latitude* is the *Perpendicular*; the *Departure* the *Base*. The *Angle* at the *Perpendicular* is the *Course*, and the *other* its *Complement*.

PROBLEM XII.

To take the *Diſtance* of any *Cape*, *Fort*, or *Iſland*, from a *Ship* at *Sea*.

Sailing W. S. W. I ſaw, at ſome Diſtance, a Point of Land, which I *ſet*, and find it bears from me W. by N. and having ſailed 6 Leagues further, I find it then bears from me N. W. by W. I would know how far this Land is from me.



33.75
33.75
16875
23625
10125
10125
1139.0625
3
3.417.1875
57.3
Nat. Rad. 60.7 is enough.

(1ſt.) Find the Perpend. BD in Triangle ABD.

N. Rad.	Op. Side AB	∠ A
As 60.7 —	18 —	33.75

18
27000
3375
60.7)607.50(10 Perpend.
607
.50

22.5
22.5
1125
450
450
506.25
3
1.518.75
57.3
Nat. Rad. 58.8 is enough.

(2d.) Find the Diſtance CB in Triangle BCD.

∠ C	Op. Side BD	Nat. Rad.
As 22.5 —	10 ——	58.8

10
22.5)5880(26.13
450
1380
1350
300
225
750
675
75

Anſwer, 26.13 Miles, the Diſtance requir'd.

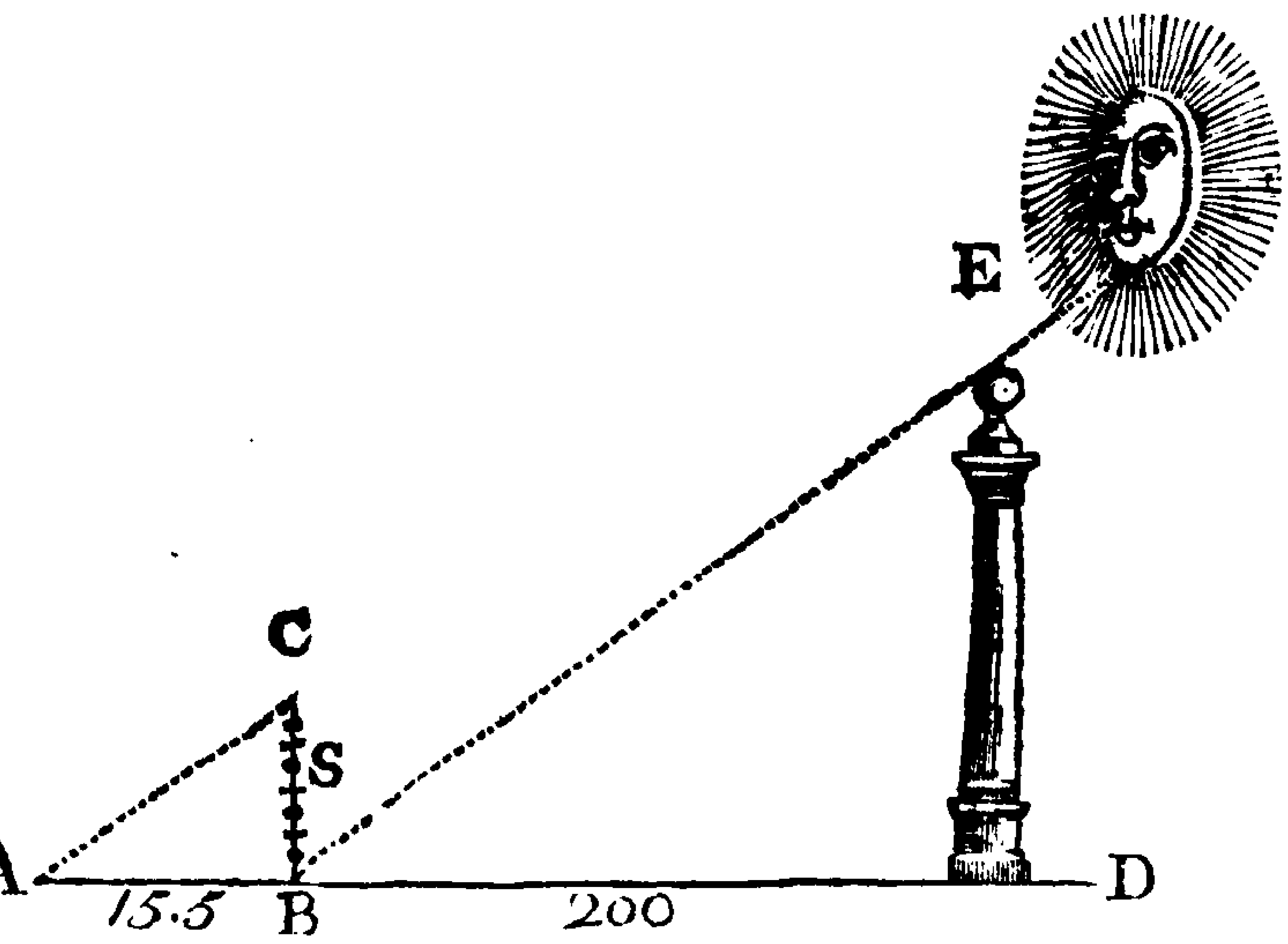
PROBLEM

PROBLEM XIII.

To take the Height of a *Tree*, *Fort*, *Obeliſk*, *Pyramid*, or any *Object*, by a *common Stick* only, when the *Sun* or *Moon* ſhines upon it.

Take a Stick of any Length, ſuppoſe 8 Feet; ſet it upright upon the Ground, as at CB in the Figure below. Mark the End of its Shadow at A, and meaſure its Length from B to A, which ſuppoſe to be 15.5 Feet. Then meaſure the Length of the Shadow of the *Pillar* or *Obeliſk* BD, which ſuppoſe to be 200 Feet. This being done, you may eaſily find the Height: For (by Reaſon of like Triangles) it will always hold,—as the Length of the Shadow of the Stick AB in the ſmall Triangle, is to its Height CB; ſo is the Length of the Shadow of the Obeliſk BD in the great Triangle, to DE the Height thereof.

```
Shad. St.  Stick.  Shad. Ob.
As 15.5  —  8  —   200
                     8
                  ————
15.5)1600.0(103.2
       155..      Height
       ———        requir'd
       ..500
         465
         ———
        .350
         310
         ———
         .40
```



In this *Manner* the Heights of the *Pyramids* in *Egypt* have been taken. Thoſe ſtupendous Buildings are ſuppoſed to have been erected by the *Children of Iſrael*, when in Bondage, for *Sepulchers* for the *Egyptian Kings*. They are the greateſt Pieces of *Antiquity* now in Exiſtence. There are ſeveral *ſmaller Ones*, but the *largeſt*, which is juſtly eſteemed one of the *Wonders of the World*, is 500 Feet in *Perpendicular Height*;—700 Feet if meaſured *obliquely* from the Bottom to the Top;—and its *Baſe* covers about 11 *Acres of Ground*.

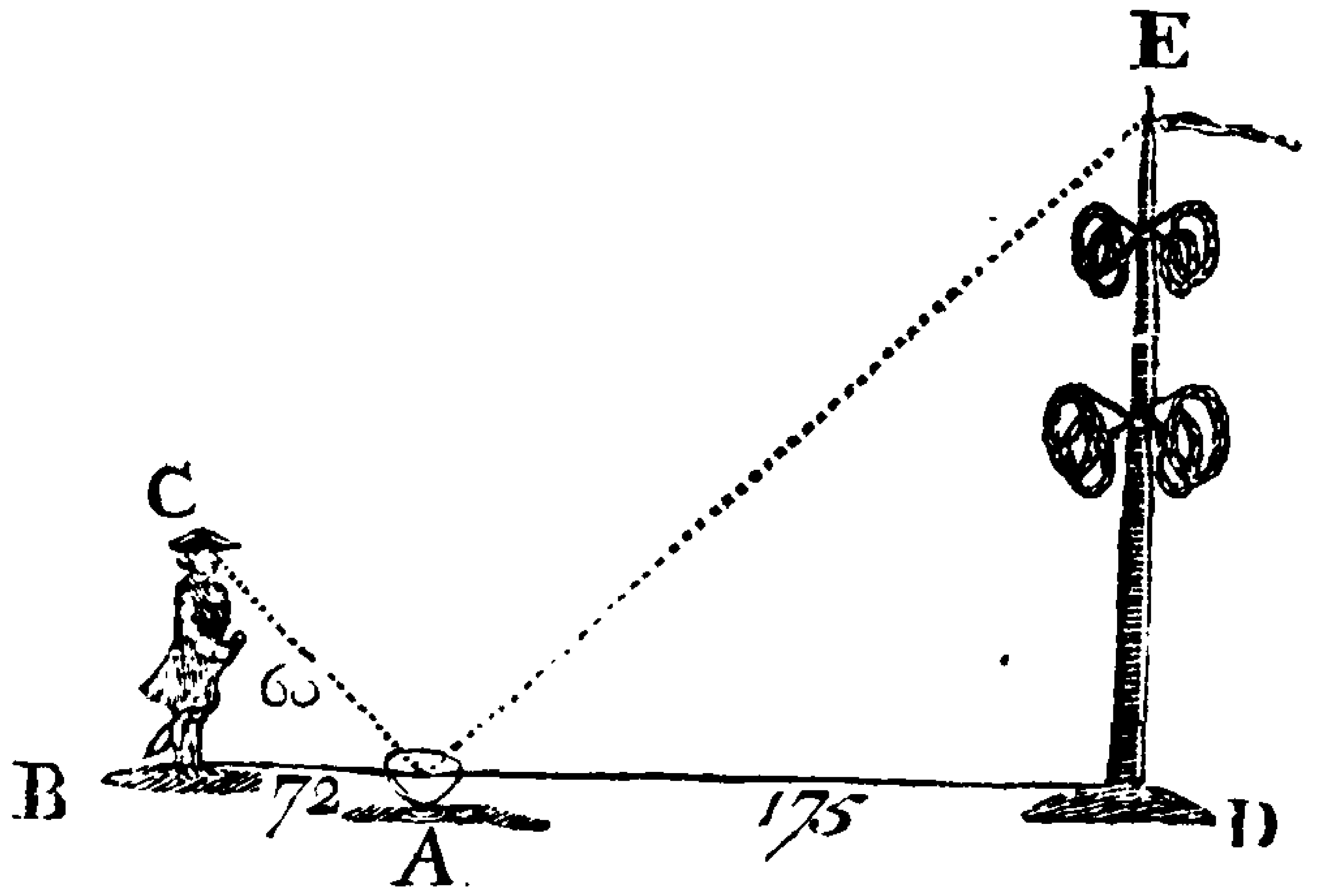
PROBLEM

PROBLEM XIV.

To take the *Height* of any *accessible Object* by a *Bason of Water*, or common *Looking-glass*.

Travelling along the Road, I see a fine *May-pole*, whose Height I would gladly know; but having no *Mathematical Instrument* with me, I procure a *Bason of Water*, which I set upon the Ground, at some Distance from the Pole, as at A; then I go backwards, till I see the Top of the Pole in the Middle of the Water, as at B. This done, I measure the Distance from my Station at B to the Bason at A, which suppose 72 Inches; and also measure from the Bason to the Bottom of the Pole at D, and find it 175 Inches. Next I measure the Height of the Eye from the Ground, which suppose 60 Inches. Then say, by the *Rule of Three*,

As the *Distance* from my *Station* to the *Bason*,	= 72	Inches.
Is to the *Height* of the *Eye*,	= 60	
So is the *Dist.* from the *Bason* to the *Foot* of the *Pole*,	= 175	
To the *Height* of the *Pole* requir'd,	= 145.8	



The same Thing may be obtain'd by a *Looking-glass*, laid truly *Horizontal*, or level on the Ground, by walking back till you can see the *Top* of the *Building*, &c. in the *Middle* of it, as was done by the *Water*.

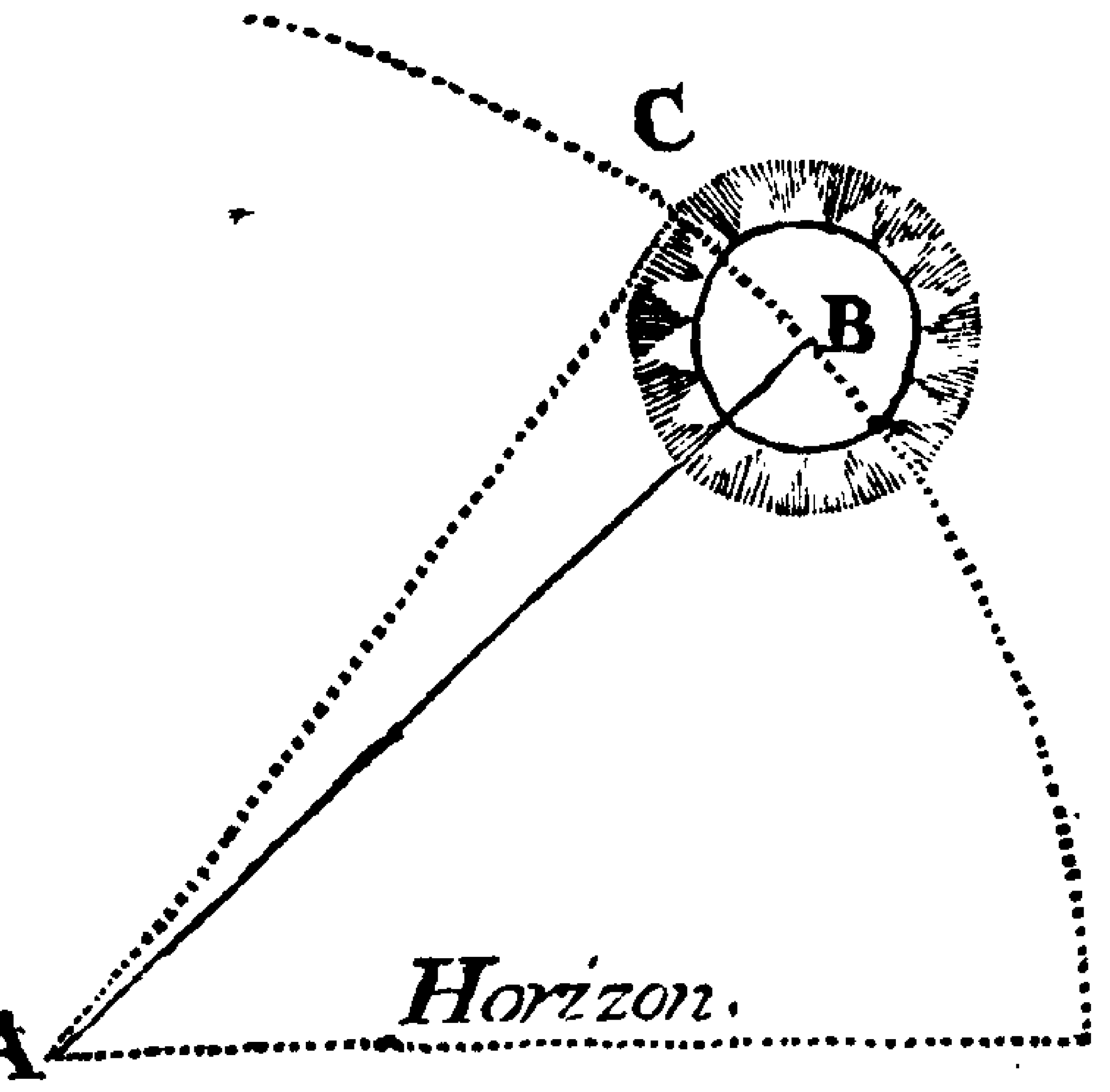
PROBLEM

PROBLEM XV.

To take the Diſtance of the *Sun*, *Moon*, or any of the *Heavenly Bodies*.

Suppoſe it was requir'd to find the Diſtance of the *Sun*, in *Diameters* of his Body from us.

With a *Quadrant* nicely graduated, take the *Altitude* of the *lower* and *upper Limb* in *Degrees* and *Minutes*, and ſubtract the one from the other; the Remainder will give the *Diameter* of the *Sun*, which we will ſuppoſe, in this Caſe, to be 32 Minutes.



Then have we given in the Triangle ABC, Right Angled at B,—the Angle at A = 16′, and the Side CB = .5 = Half the Diameter of the Sun, whoſe whole Diameter we will call 1; to find the Diſtance, or Side AC.

Ang. A : Side BC :: Nat. Rad.
As .26 —— .5 —— 57.3
.5
.26)2865(110 Diameters; and ſo far is the *Sun* of its own *Breadths* from us in the *Winter* *, but in *Summer*, the Angle being a little ſmaller, he muſt, conſequently, be a little further from us.
26
——
26
26
——
05

Having found the *Diſtance* of any *Heavenly Body* in its own *Diameters* from us; you may eaſily tell its *Diſtance* in *Miles*, if you firſt know the *Diameter* of that Body in *Miles*. For, the Diſtance in Diameters, multiply'd by the Miles in one Diameter, gives the Diſtance ſought.

For the Uſe of the Learner, I have here ſubjon'd a Table of the *Diameters* of all the *Planets* in *Engliſh Miles*; whoſe *Diſtances* he may calculate at his Leiſure.

Sun 800.000—*Mercury* 2460—*Venus* 7906—*Earth* 7964—*Mars* 4444 *Jupiter* 81.155—*Saturn* 67.870—*Moon* 2175.

* In this Manner we can tell the *apparent* Diſtance of any of the *Heavenly Bodies*: For the *Sun* appearing about 1 *Foot* in Diameter, his *apparent* Diſtance can be only 110 *Feet*, or 37 *Yards*. The *apparent* Diſtance of the *Moon* is nearly the ſame.

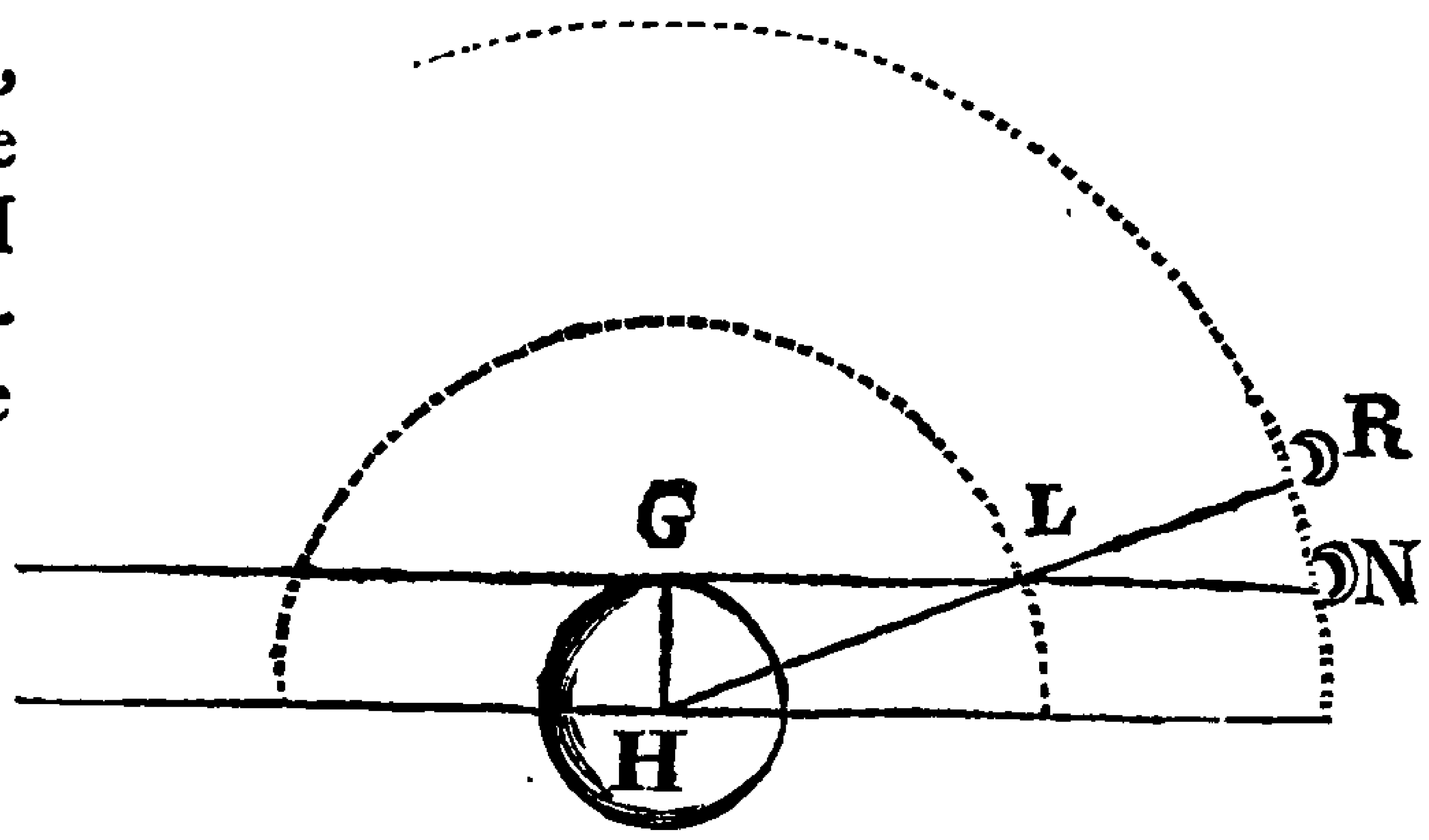
PROBLEM XVI.

To find the *Distance* of the *Moon* from the *Earth* another Way.

As the Places of all the *Heavenly Bodies* are computed from the *Center* of the *Earth*, but we are obliged to view them from the *Superfices*, they must therefore appear something lower in the Heavens than they really are.

Thus, suppose, for Instance, the ☽ to be at L in the visible Horizon; an Observer at G will see her in the Line GN, but an Eye in the Center will see her in the Line HR.—The former is her *apparent Place*, known by Observation with exact Instruments; the latter her *true Place*, and known by *Calculation* from *Astronomical Tables* of her Motion. The Difference between these two Places is the Measure of the Angle GLH, which is called the *Horizontal Parallax*. This Angle has been found, when the ☽ was at a *mean Distance*, to be 57 Minutes nearly.

Then in the Triangle HGL, Right Angled at G, we have the Angle at L = 57′, the Side GH = the Earth's *Semidiameter* given; to find HL, which is done thus.



Ang L : Semid. GH :: N. Rad.
As .95 — 1 — 57.3
1

.95)57.30(60 Semidiameters: which multiply'd by the Earth's Semidiameter=4000 nearly, you have 240000 Miles the Distance of the Moon sought.
570

30

When the ☽ is at her *least* or *greatest* Distance from the Earth, she will be about 3 *Semidiameters* of the Earth *nearer* or *farther off*, than at her *mean* Distance.

In like Manner may the Distance of any of the *Planets*, *Comets*, or other *cælestial Phænomenon* be determined, by obtaining its *Parallax* in the *Horizon*.

NOTE. To find the *Distance* of any Place to which the *Sun*, *Moon*, or any *Star* is *vertical*, i. e. over its Head, or in its *Zenith*. See my *Geography*, p. 30.

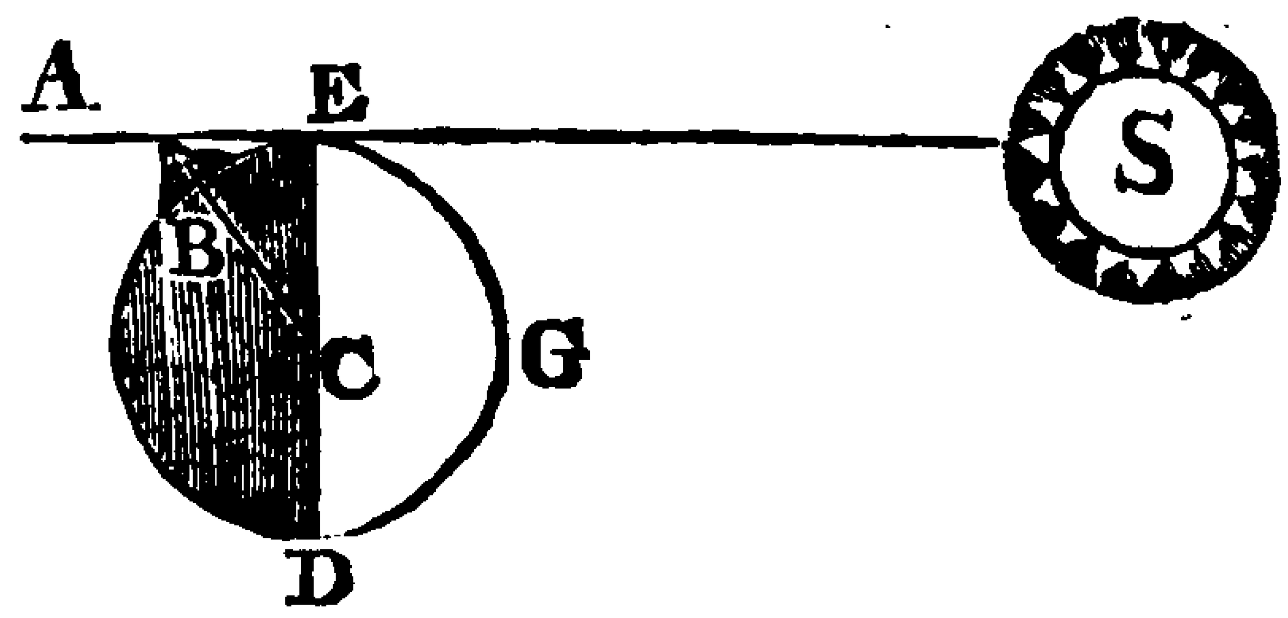
PROBLEM

PROBLEM XVII.

To measure the *Height* of a *Mountain* in the *Moon.*

The *Moon* is suppos'd to be compos'd of *Land* and *Water* as our Earth is; consequently there must be some Unevennesses, or Inequalities, of *Hills* and *Vallies* as here. This indeed is confirmed by viewing her through a good *Telescope*; for then we find, that the Line, which seperates the Light from the Dark Parts on her Surface is not even or regular, but tooth'd and jagg'd with innumerable Breaks; and even in the dark Parts, near the Borders of the lucid Surface, there are seen some small Spots enlighten'd by the Sun, which are very visible when the ☽ is three or four Days old, and which can be nothing else but the Tops of *Mountains* or *Rocks*; since it is impossible for the Sun's Rays to fall upon those Parts only, unless they were higher t' an the Rest of the Surface.

The *Lunar Mountains* are found to be *higher*, in Proportion to the Body of the ☽ than any Hills upon our Globe.—The Manner of calculating their Heights is this.



Let EGD be the Surface of the ☽ and ECD the Diameter of the Circle bounding *Light* and *Darkness*. A the Top of a Hill within the dark Part, when it first begins to be illuminated by a Ray of Light coming from the Sun at S. Then observe with a Telescope the Proportion of the Right Line AE, (*i. e.* the Distance of the Point A from the lucid Part) to the Diameter (or Semidiameter) of the ☽ ED, for that being ascertain'd, you have in the Triangle AEC, Right Angled at E, (where the Ray of Light touches the ☽) the two Sides AE and CE, to find the *Hypothenuse* AC, from which subtracting BC = EC, there will remain AB the Height of the *Mountain.*

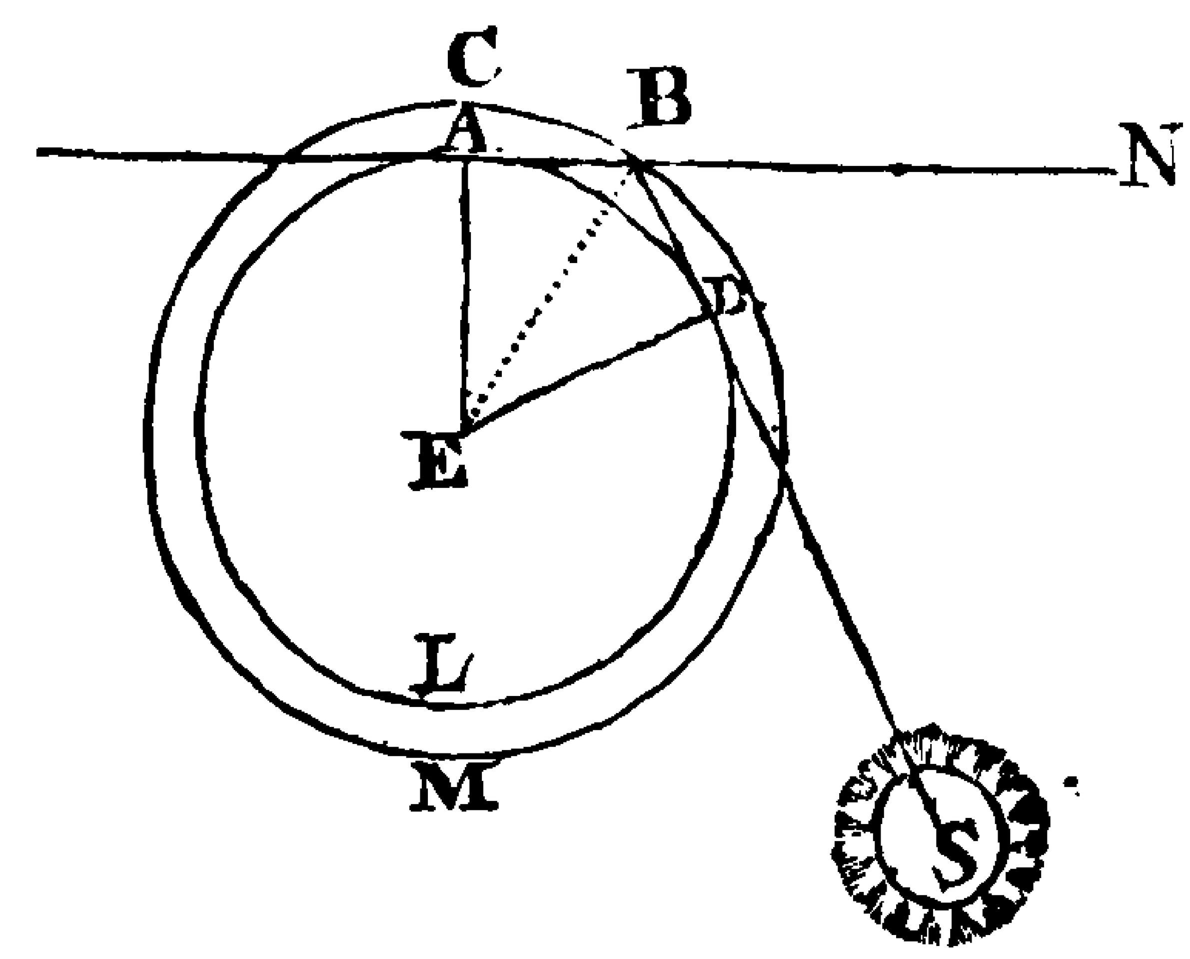
Ricciolus, on viewing the ☽ when about four Days old, observ'd the Top of a Hill called *Saint Catherine*, near the N. Part of *Mount Taurus*, (see my *Astronomy)* to be illuminated, and that it was then distant from the Surface about $\frac{1}{8}$ of the Moon's *Semidiameter*. Now as the *Semidiameter* of the ☽ ECis about 1088 Miles, the Line AE being $\frac{1}{8}$ of it, must be = 136 Miles. Consequently, if the □ of EC and □ of AE be added together, and then the □ Root of it be extracted, it will give the Line AC, from which subtracting the Moon's *Semidiameter* BC or CE, the Remainder, which is 8 Miles will be the Height of the *Mountain* sought.

PROBLEM

PROBLEM XVIII.

To meaſure the *Height* of the *Atmoſphere.*

The *Atmoſphere* is that Circle of *vaporous Air* ſurrounding the Earth, which being illuminated by the *Sun*'s Rays makes the Brightneſs and Glory of the *Firmament* we behold, whilſt the Sun continues above the *Horizon*. And, after the Sun is gone down, the *Atmoſphere*, which is higher than we are, will ſtill continue to be illuminated by thoſe Rays paſſing by the Earth over our Heads; but this *Brightneſs* grows *leſs* and *leſs* as the Sun deſcends lower, till he arrives at 18° below the *Horizon*; when all the Parts of the Air above fall out of his Rays, and, conſequently, become dark.



To make this plainer; ſuppoſe the inner Circle ADL repreſents the Earth, and the Circle CBM the *Atmoſphere*. Suppoſe a Perſon ſtanding upon the Earth at A whoſe *ſenſible Horizon* is AN. Alſo let SB be a Ray of Light coming from the *Sun*, touching the Earth at D, which falls upon the diſtant Part of the Air, in the Horizon at B, at which time *Twilight* ceaſes. This has been found to happen when the Sun is deſcended 18° below the *weſtern* Horizon in the *Evening*; and alſo when he is approached within 18° of the *eaſtern* Horizon in the *Morning*.

Now as the Arch AD is 18°, we have, by drawing the Line EB, two equal Triangles, from either of which we may find the Height of the *Atmoſphere* requir'd.——For in the Triangle ABE, Right Angled at A, we have given the Side AE the *Earth*'s *Semidiameter*, and the Angle AEB = 9° = half the Arch of the *Sun*'s Deſcent below the *Horizon*, and the Angle ABE = 81°, to find the *Hypothenuſe* EB, from which if you ſubtract the *Semidiameter* of the Earth, the Remainder will be the Height of the *Atmoſphere*.

Thus, Ang. B : Earth's Semid^r. AE :: N. Rad. : Hypoth. EB
As 81 ——— 4000 ——— 81.99 —— 4048
4000 Earth's Semidiameter ſubtract

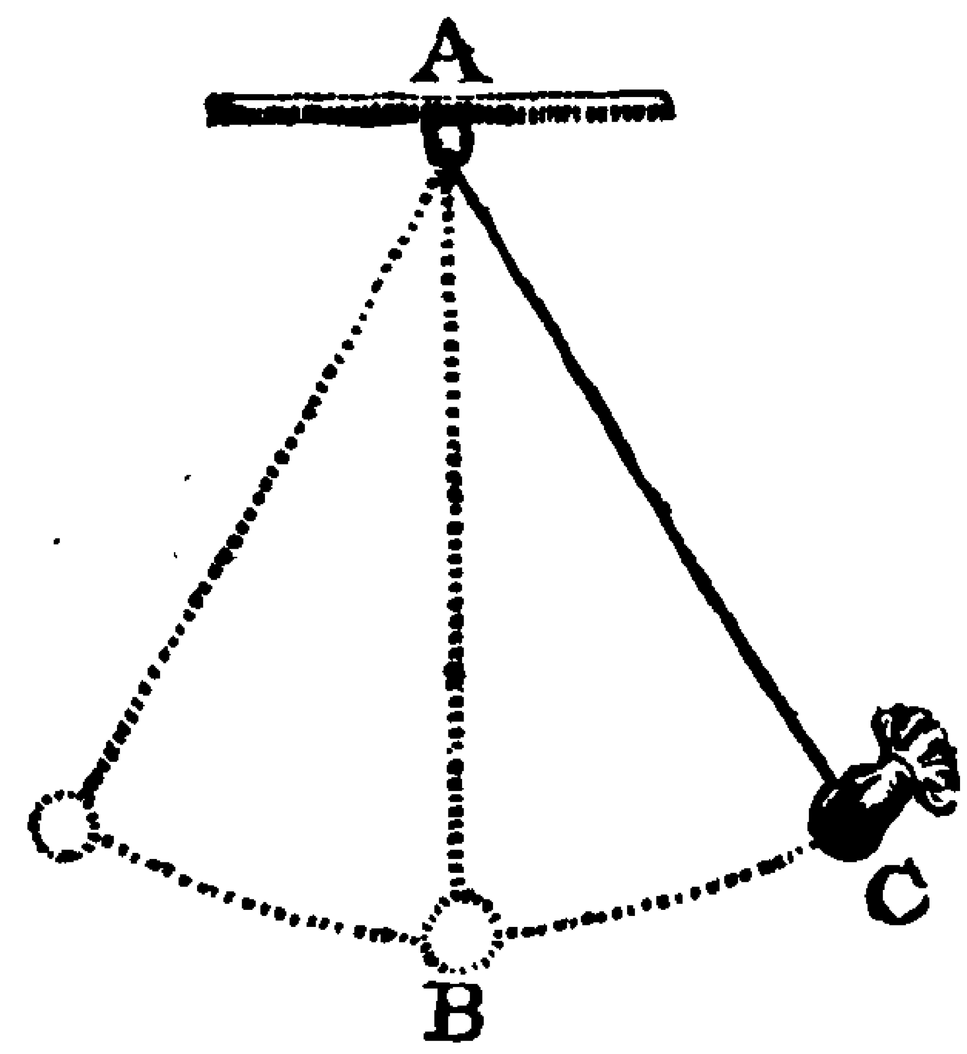
48 Height of the Atmoſphere requir'd.

PROBLEM

PROBLEM XIX.

To meaſure the *Diſtance* of a *Cloud,* from which iſſues *Lightnings* and *Thunder.*

Take a ſmall *Ball* of *Lead, Ivory,* or any other matter, and affix it to the End of a fine *Thread.* Then meaſure from the Center of the Ball, along the Thread, exactly 39.2 *Inches,* where make a *Loop.* This done, ſuſpend it by that Loop to the Ceiling of the Room, or to any other Place where it may hang freely, and vibrate backwards and forwards like a *Pendulum,* as in this *Figure.* Now the Property of this little Inſtrument is, that *each* Vibration, whether it paſſes through a larger or a ſmaller Space, will be performed in *one Second* of Time.



Being thus prepar'd, take the Ball in your Hand, and drawing it aſide from its perpendicular Direction AB, to any Diſtance, ſuppoſe to C, hold it there till you ſee the flaſh of *Lightning* paſs by, at which Moment let it go, and count the *Number of Vibrations* till you hear the Stroke of the *Thunder.* Then theſe Vibrations multiplied by 1142, (the Number of Feet, *Sound* uniformly paſſes thro' in each *Second)* the Product will be the Height of the *Cloud* in Feet, if it *be nearly over the Place where you are*; or its Diſtance from you in any other *Situation.*

Thus, ſuppoſe the *String* is found to make 8 Vibrations between the *Lightning* and the *Thunder*; then $8 \times 1142 = 9136$ Feet, which, divided by 5280 (the Feet in 1 Mile) gives 1¾ Mile nearly; and ſo far is that alarming Tempeſt from you.

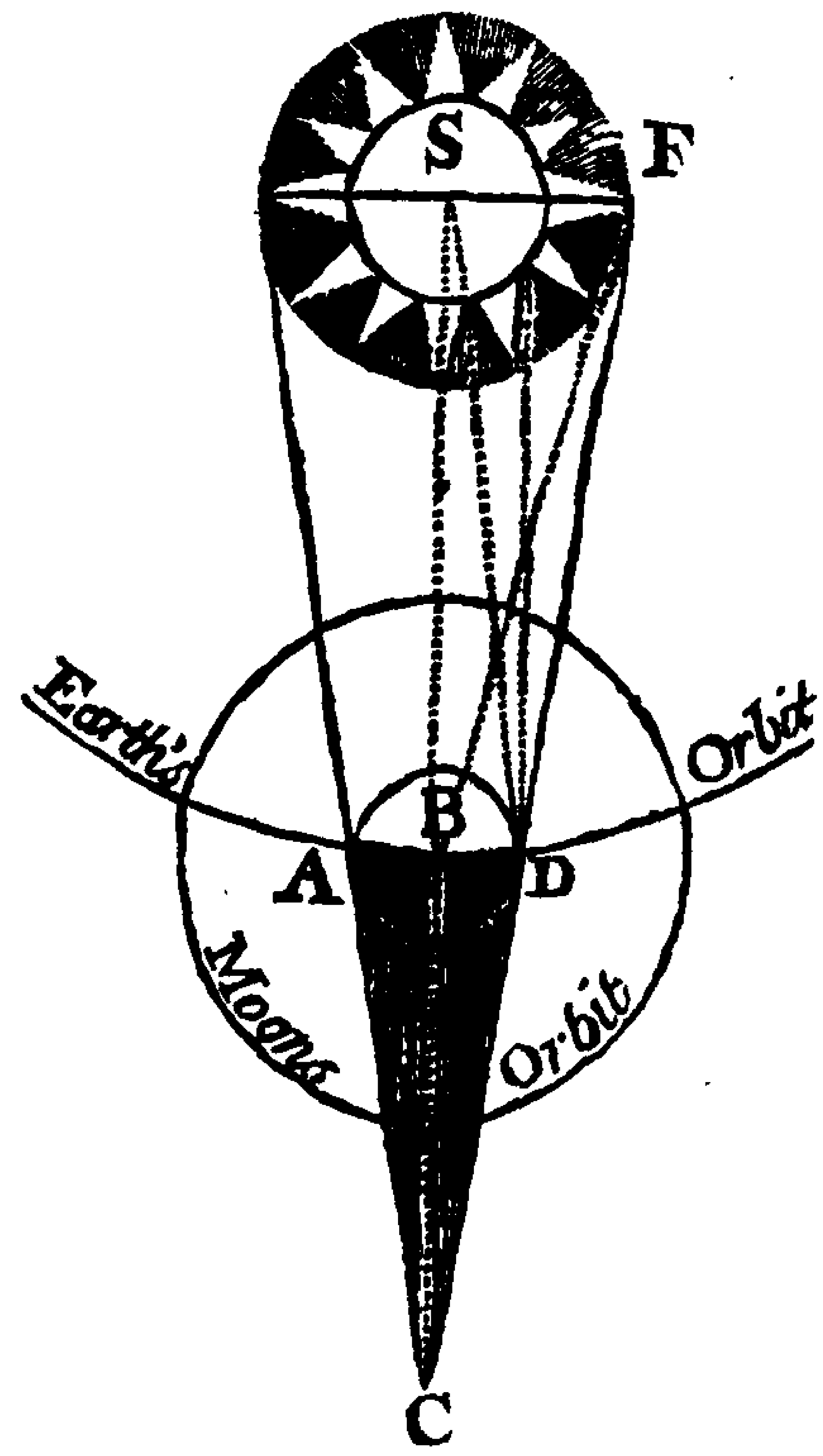
In this Manner you may continue to meaſure the Diſtance of the *Cloud* all the Time it paſſes from your *Zenith* to the *Horizon,* and by that Means be acquainted with the *Danger* it ſeems to threaten the *Neighbourhood,* as well as the *Extent* of the *viſible Hemiſphere* of *Clouds.*

The *Diſtance* alſo of a *Ship* at *Sea,* or a *Fort,* may be eſtimated in the ſame Manner, by counting the Vibrations from the *Flaſh of the Powder* to *the Report of the Gun.*

PROBLEM XX.

To calculate the *Length* of the *Earth*'s or *Moon*'s *Shadow*.

The Angle of the *Cone* ACD of the *Earth*'s *Shadow* in the annex'd Figure, is equal to the *Sun*'s apparent *Diameter* *, which, at a mean Diſtance from us, is about 32 *Minutes.*—Hence, in the Triangle ACB, Right Angled at B, we have the Angle ACB = 16 *Minutes*, the apparent *Semidiameter* of the *Sun*, and AB the *Semidiameter* of the *Earth* = 1; to find AC or BC the Length of the *Shadow*, which is done thus.



$\angle$ ACB : Side AB :: N. Rad.
As .266 — 1 —— 57.3
1

.266)57.300(215 Semidiameters = AC
532
410
266
1440
1330
110

Thus, when the *Sun* is at a *mean* Diſtance from us, the *Shadow* of the *Earth* reaches about 215 *Semidiameters* beyond it: But when the *Sun* is at his *greateſt* or *leaſt* Diſtance, the *Shadow* will be *lengthened* or *ſhortened* 3 or 4 *Semidiameters*, more or leſs.

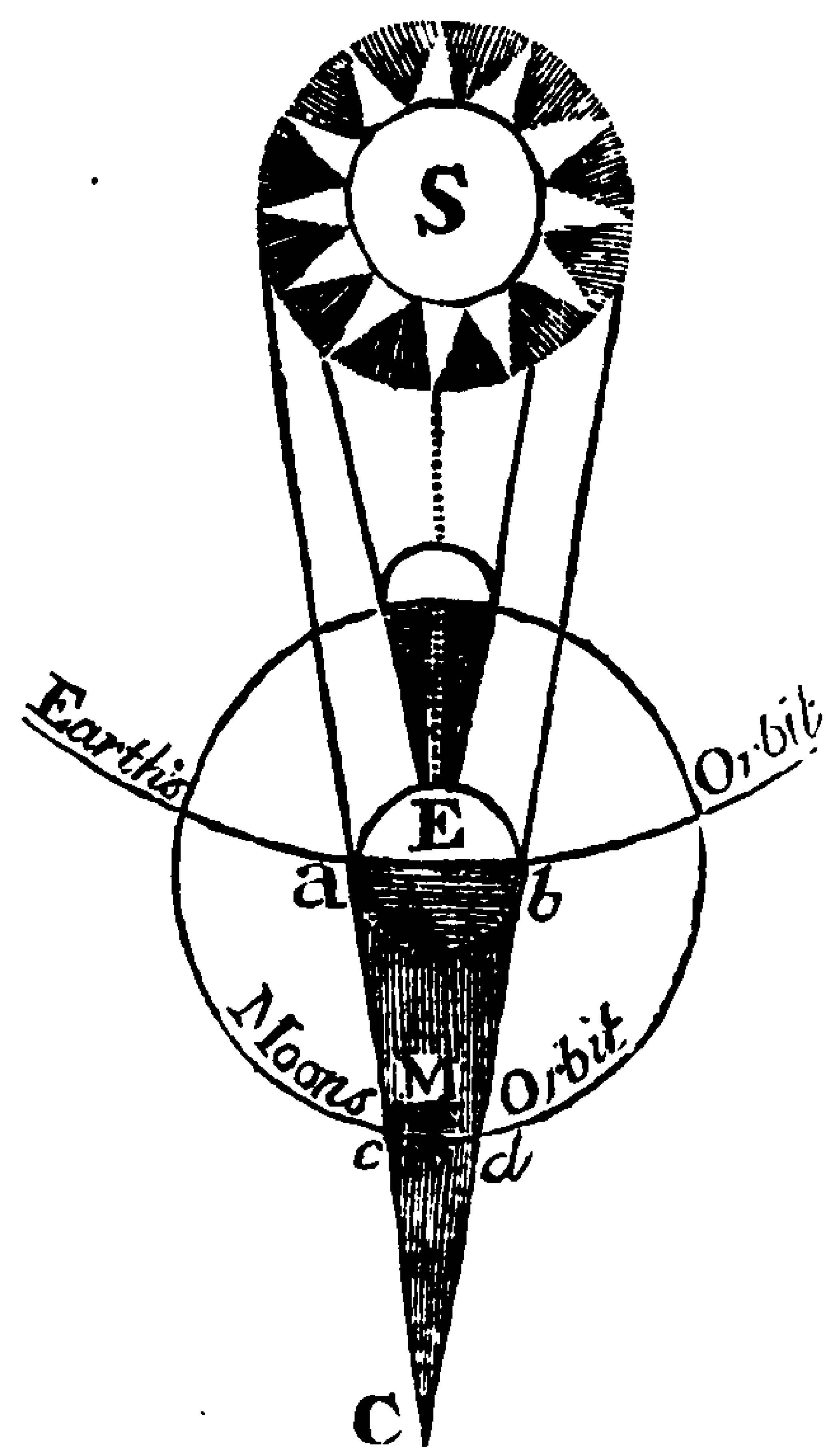
Hence, you may alſo determine the *Height* of the *Moon*'s *Shadow*: For, as the *Moon* is never at any great Diſtance from the *Earth*, the apparent *Semidiameter* of the *Sun* muſt be nearly the ſame *there* as *here*. Conſequently, the *Moon*'s *Shadow* muſt contain the ſame Number of *Semidiameters* of the *Moon*, as the *Earth*'s *Shadow* does *Semidiameters* of the *Earth*: Which *Semidiameters*, multiply'd by the *Miles* in the *Semidiameter* of the *Moon* or *Earth*, will give the Length of the *Shadow* reſpectively in Miles.

* The *Semiangle* of the *Cone* of the *Earth*'s *Shadow* BCD, is equal to the apparent *Semidiameter* of the *Sun* view'd from the Top of the Shadow, which Angle is always equal (in the Shadow of every Planet) to the apparent *Semidiameter* of the *Sun* SBF, leſſen'd by his *Horizontal Parallax* BSD at that Planet. But as the *Horizontal Parallax* of the *Sun*, i. e. the Angle under which the *Earth* is ſeen from thence, is ſcarcely 10 *Seconds*, it may be omitted, as is done in the above Calculation.

PROBLEM XXI.

To calculate the *Diameter* of the *Earth*'s *Shadow* at the Diſtance of the *Moon*; and alſo, the *Diameter* of the *Moon*'s *Shadow* at the *Earth*.

In the following Figure let S repreſent the *Sun*, E the *Center* of the *Earth*, M the *Moon*, EC the *Cone* of the *Earth*'s *Shadow* (at a mean) = 215 *Semidiameters* of the *Earth*: Then MC will be the *Cone* of the *Earth*'s *Shadow* reaching beyond the *Moon*, whoſe Length is thus found.



	Semid^rs.
From EC the Cone of the Earth's Shadow	= 215
Subtract EM the Diſt. of the Moon in Earth's Semid. =	60*
Remains MC the Shad. of the Earth beyond the Moon =	155

Then, by Reaſon of ſimilar Triangles, it will always hold;

As the Length of the whole Shadow	EC
Is to the Diameter of the Earth	ab
So is the Length of the Shadow beyond the Moon	MC
To the Diameter of the Shadow at the Moon	cd.

```
   EC   :   ab   ::   MC
As 215  —  7964  —   155
            155
           -----
           39820
           39820
           7964
          -------
215)1234420(5741 Miles = cd, the Diameter of the Earth's Shadow at the Diſtance of the Moon.
    1075
    ----
    1594
    1505
    ----
      892
      860
      ---
       320
       215
       ---
       105
```

By this Problem alſo may be found the *Diameter* of the *Moon*'s *Shadow* at the Surface of the Earth, and, conſequently, how much of the Earth is involv'd in that *Shadow* in an *Eclipſe* of the *Sun*. For the *Length* of the *Moon*'s *Shadow* is found to be about 60 *Semidiameters* of the Earth, which is nearly the *Moon*'s *mean* Diſtance from us; her *Shadow*, in that State, muſt, therefore, reach as far as the Center of the Earth.—But as the *Moon* is, ſometimes, almoſt 4 *Semidiameters* of the Earth *nearer*, the *Shadow* muſt reach 4 *Semidiameters* beyond the Center of the Earth; and when the *Moon* (as ſhe ſometimes is) is 4 *Semidiameters further* from us, the *Shadow* will then not reach the Earth at all. In ſuch Caſe, the *Sun*, though centrally *eclipſ'd*, will not be *totally* covered by the *Moon*; but an *Annulus*, or Ring of Light, will appear round the Border of that Luminary, as happen'd April 1ſt, 1764.

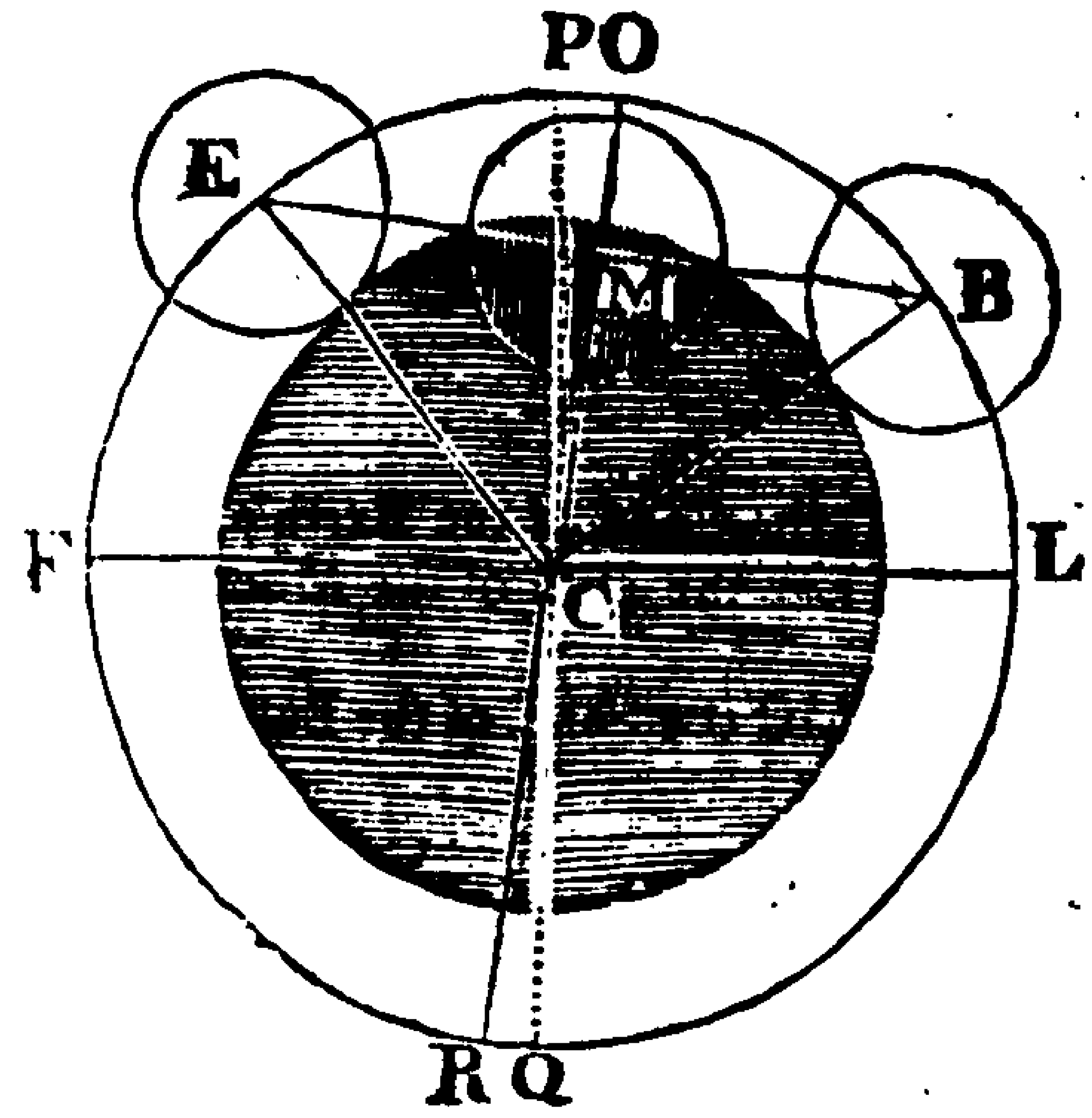
* The Method of finding the *Diſtance* of the *Moon*, in Semidiameters of the Earth, is ſhewn at Problem XVI.

PROBLEM

PROBLEM XXII.

To calculate the *Beginning*, *End*, and *total Duration* of an *Eclipse*.

Having, from *Astronomical Tables*, obtain'd the *Time* of the *Middle of the Eclipse*, which suppose to be December 21 Day, 11 Hours, 49 Minutes, with the *Latitude* of the *Moon* at that Time, = 40 Minutes, you may then proceed to find the *Beginning*, *End*, and *total Duration*, as follows.



From a Scale of equal Parts, of any Size, take off the *Semidiameter* of the the *Earth*'s Shadow, which, at the Distance of the ☽ (at a Mean) is about 42′; and, setting one Foot in C, describe the inner *shaded Circle*, to express that Part of the Cone of the Earth's Shadow cut off at that Place where the ☽ passes thro' in that *Eclipse*.—With the *Sum* of the *Semidiameter* of the ☽ = 16′, and *Earth*'s Shadow = 42′, (which together = 58′) taken from the same equal Parts describe the outer Circle.—Draw the Line FL through the Center, to represent the *Ecliptic*, or Path of the *Earth*'s Shadow; cross it, at Right Angles, with the dotted Line PQ, to express the *Poles* of the *Ecliptic*.—Then with a *Line of Chords*, or a *Protractor*, set off $5\frac{1}{2}°$ from P, upon the outer Circle, towards the *Right Hand*, because the *Latitude* of the ☽ is *North ascending*, to express the Angle of the *Moon*'s *Path* with the *Ecliptic*, and draw the Line OR.—Take the *Latitude* of the ☽ = 40′, from the same Scale of equal Parts, and set it from C to M upon the Line CO.—Then draw a Line through M, at Right Angles to CO, and that Line will represent the *Path* of the ☽ during the *Eclipse*.—Next, with the *Semidiameter* of the ☽ = 16′, taken from the equal Parts, describe, on the three Points B, M, and E, severally, the three *little Circles*; so will the Circle at B represent the ☽ at the *Beginning*, that at M the *Middle*, and that at E, the *End* of the Eclipse.

Now, from the Center C, draw two Lines to B and E; then in the *Right Angled Triangle* CMB, Right Angled at M, we have given CM the *Latitude* of the ☽ = 40, and CB = CE the *Sum* of the *Semidiameters* of the *Moon* and and *Earth*'s *Shadow* = 58; to find MB = ME, the *Motion* of *Half Duration* of the *Eclipse*.

From Square of 58 = 3364
Take Square of 40 = 1600

Extract the Root 1764(42 *Minutes*, = the Motion of *Half* the *Duration*.
16
82)164
164

And because the Moon passes over 31 of these Minutes (at a mean Rate) in one Hour, we have this Proportion.——If 31 Min. : 1 Hour :: 42 Min. : 1 Hour 21 Min. which *subtracted from*, and *added to*, the Middle, will give the *Beginning* and *End*.

		D.	H.	M.
Middle of the Eclipse	Dec.	21	11	49
Half Duration subtract and add			1	21
Beginning		21	10	28
End		21	13	10
Total Duration			2	42

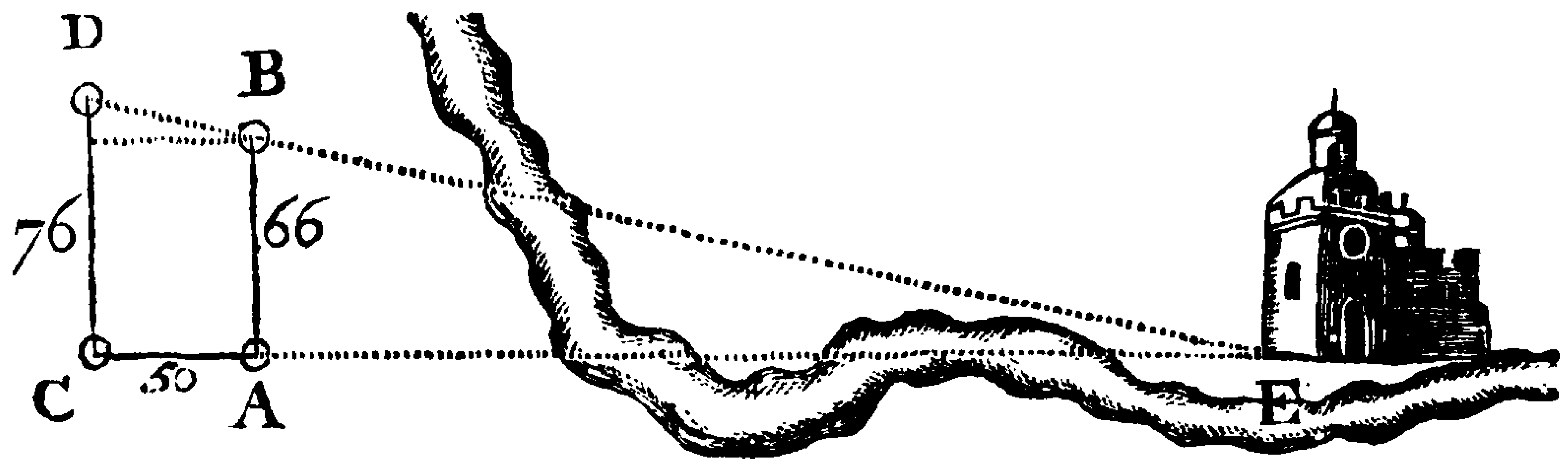
NOTE. The Middle of an *Eclipse*, with the *Lat.* of the *Moon*, may be easily had from my *Astronomy*.

PROBLEM

PROBLEM XXIII.

To take the *Diſtance* of an *inacceſſible Object* without the Help of *any Inſtrument.*

Suppoſe E, in the following Figure, to be a *Fort,* whoſe *Diſtance* you want to know, and you cannot approach it, on Account of ſome *Moat, Ditch,* or *River,* lying between you and the *Object.*



Firſt, at ſome Diſtance from the *Ditch* or *River,* ſet up a *Stick,* as at C; then advance, in a Right Line, towards E, any Number of Yards, ſuppoſe 50, and ſet up another Stick at A; next, move, in a Line *perpendicular* to CE, from A to B, any Diſtance, ſuppoſe 66 Yards, and ſet up another Stick at B; then return back to C, where you began, and remove from thence, in a Line *perpendicular* to CE, till you ſee the Stick at B and the Object E in a Right Line, and ſet up another Stick in that Place at D, meaſuring the Diſtance from C to D, which ſuppoſe 76 Yards. Then it will always hold;—

$$\left.\begin{array}{lcr} \text{As the Difference between AB and CD} & = & 10 \\ \text{Is to the Length between A and C} & = & 50 \\ \text{So is the Diſtance CD} & = & 76 \\ \text{To the Diſtance between C and E} & = & 380 \end{array}\right\} \text{Yards.}$$

NOTE. If, in the *third Term,* you had us'd the *Diſtance* AB = 66 Yards, you would have obtain'd the Diſtance from A to E = 330.

PROBLEM XXIV.

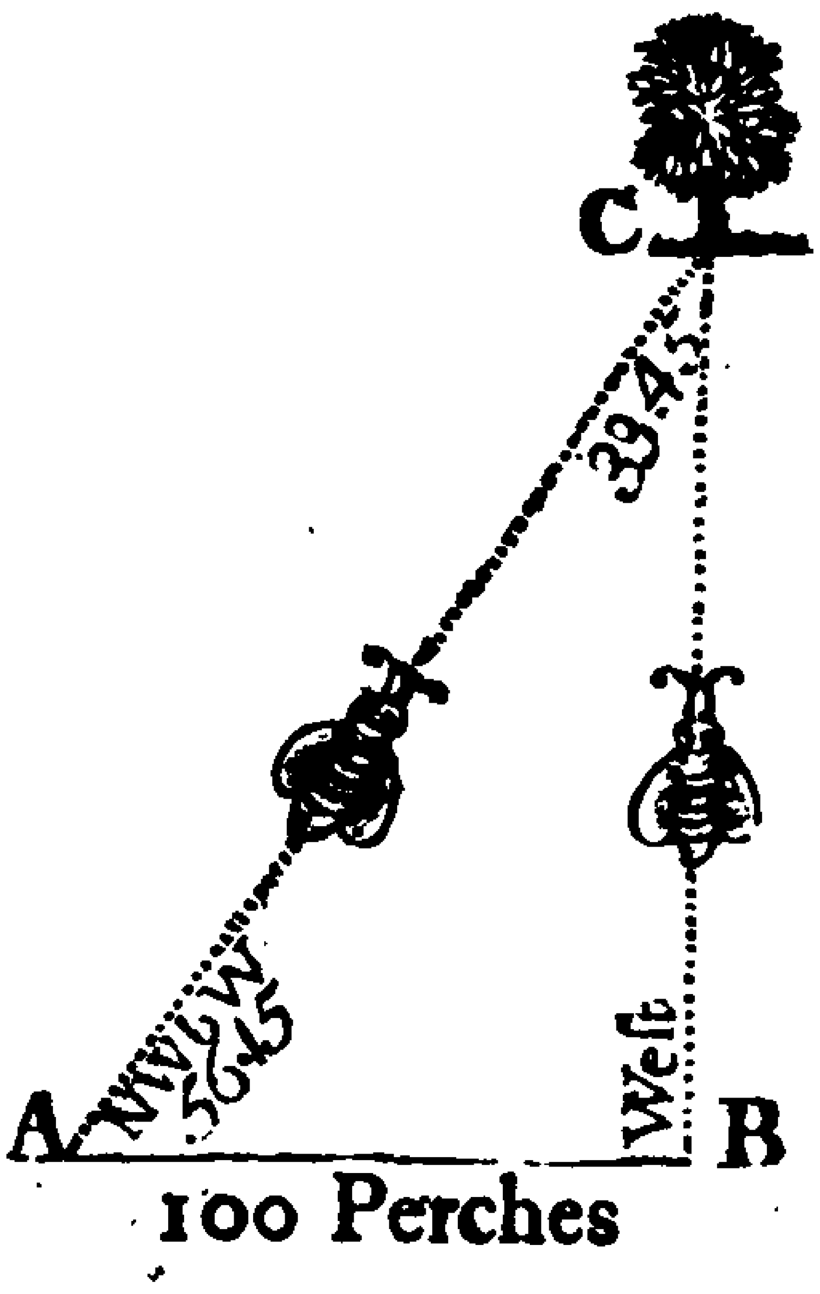
To find, by a *new Method*, where the *Bees* hive in large and extenſive *Woods*, in Order to obtain their *Honey*.

Take a *Plate* or ſmall Piece of *Board*, on which is ſpread a little *Honey* or *Treacle*, and ſet it down on a *Rock* or Stump of a *Tree* within the *Wood*. This the *Bees* will ſoon find out if any are near, for it is generally believ'd they ſmell Things of that Nature at the Diſtance of a Mile, or further. Whilſt theſe little *Creatures* are feeding, ſecure *two* or *three* of them in a Box, or ſomething convenient. Then let one of them go, obſerving carefully, by a *Pocket Compaſs*, the Courſe he takes; for *Bees*, after they riſe in the *Air*, fly directly in a ſtrait Line to the *Tree* where their Hive is.

Suppoſe, for Example, the *firſt Bee* is found to fly directly *Weſt*; then you may be ſure the *Tree* is ſome where in that Line from your preſent Station. But, in order to know how far, you muſt make an *Offset*, either *North* or *South*, as large as you can, which in this caſe we will ſuppoſe to be 100 *Rods* or *Perches* (the larger the better) to the *South*. Here you muſt let go another *Bee*, obſerving his Courſe as before, (for this Bee, being loaded like the other, will fly *directly* to the *Hive*) which Courſe we will ſuppoſe to be N. W. by W. —56° 15′ towards the *Weſt*, it only remains now to find where theſe two Courſes or Lines interſect or meet with each other, for there you will find the Tree in which the *Honey* is.

This may be eaſily done.—For in the *Right Angled Triangle* ABC, are given the Right Angle B, the *Courſe* of the firſt Bee, the Angle at A, the *Courſe* of the ſecond Bee, and the Diſtance AB; to find BC, or AC, the Diſtance of the *Tree* from either Station.

	∠C	:	Baſe	::	N. Rad.	:	Diſt. AC
As	33.75	—	100	—	60.7	—	179.8 Perches.



Formerly, they found the *Honey* by ſurprizing the Bees, and following them, one after another, till they found out the Hive; but ſince this *Trigonometrical Method* has been us'd, the Searchers diſcover that Booty in a few Hours, which before requir'd many Days.

CONCLUSION.

CONCLUSION.

THESE few *Problems* are ſufficient to point out the great *Uſe* of this Branch of *Learning*. The Advantages reſulting from it to Society are very great;—almoſt infinite.——Nothing however poſited in the *Heavens*;—nothing upon the *Earth* or *Seas*;—but its *Diſtance* and *Dimenſions* may be aſcertained by it.——It is no Wonder then, that *Pythagoras*, a learned Philoſopher of *Samos*, when he had diſcover'd that famous *Propoſition* (47th of 1ſt Book of *Euclid*) which is the *Foundation* of this *Science*, ſhould, in Gratitude, ſacrifice an *Hecatomb*, i. e. 100 Oxen, to the *Muſes*, for inſpiring him with ſuch an uſeful Invention, which he judg'd beyond the Power of *human Abilities* to diſcover.

Thus by one plain *Geometrical Figure*, having three Sides and three Angles, and aſſiſted by the *Rule of Three*, you ſee what amazing *Truths* may be diſcover'd. This illuſtrates not only the old Motto,—*Tria ſunt omnia*—but aſo proves the *Truth* of Mine in the *Title-page*.

Cuncta Trigonus *habet, ſatagit quæ docta* Matheſis,
Ille *aperit clauſum quicquid* Olympus *habet*.

Which may be Engliſh'd thus;

In Heaven *the* latent Science *lay conceal'd*
Till the Triangle *came, and* Truth *reveal'd*.

FINIS.

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